# SEM II GEOACORE04T TOPIC: LOGARITHM

**"Logarithm"** is a word made up by Scottish mathematician John Napier (1550-1617), from the Greek word *logos* meaning "proportion, ratio or word" and *arithmos* meaning "number", ... which together makes "ratio-number".

**Logarithm is the exponent or power to which a base must be raised to yield a given number. The power is sometimes called the exponent.**
In other words, if $b^y = x$ then $y$ is the logarithm of $x$ to base $b$ and $x$ is the number. For example, if $2^4 = 16$, then 4 is the logarithm of the number16 with the base as 2. We can write it as $4 = \log_2 = 16$.

The **exponent** says **how many times** to use the number in a multiplication.

In this example: $\mathbf{2^3 = 2 \times 2 \times 2 = 8}$

*(2 is used 3 times in a multiplication to get 8)*

- "the logarithm of 8 with base 2 is 3"
- or "log base 2 of 8 is 3"

Notice we are dealing with three numbers:

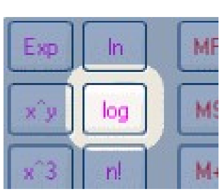
- the **base**: the number we are multiplying ( "2" in the example above)
- how often to use it in a multiplication (3 times, which is the **logarithm**)
- The **number** we want to get (an "8")

**Exponents and Logarithms are related**...For example, $2^3 = 8$; therefore, 3 is the logarithm of 8 to base 2, or $3 = \log_2 8$. In the same fashion, since $10^2 = 100$, then $2 = \log_{10} 100$.

Broadly two types of logarithms:

**Common Logarithms: Base 10**

Sometimes a logarithm is written **without** a base, like this: log(100)
This ***usually*** means that the base is really 10.
It is called a "common logarithm"

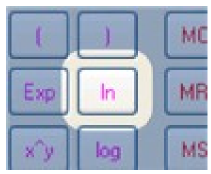


On a calculator it is the "log" button.

Logarithms of the latter sort (that is, logarithms with base 10) are called common, or Briggsian, logarithms and are written simply log *n*.
Invented in the 17th century to speed up calculations, logarithms vastly reduced the time required for multiplying numbers with many digits. They were basic in numerical work for more than 300 years, until the perfection of mechanical calculating machines in the late 19th century and computers in the 20th century rendered them obsolete for large-scale computations.

## Natural Logarithms: Base "e"

Another base that is often used is e (Euler's Number) which is about 2.71828. This is called a "natural logarithm". Mathematicians use this one a lot.

On a calculator it is the "ln" button. 


It is how many times we need to use "e" in a multiplication, to get our desired number.

Example: **ln(7.389) = $\log_e$(7.389) ≈ 2**

Because **$2.71828^2 \approx 7.389$**

The natural logarithm (with base $e \cong 2.71828$ and written ln *n*), however, continues to be one of the most useful functions in mathematics, with applications to mathematical models throughout the physical and biological sciences.

### Logarithmic Table

It is not always necessary to find the logarithm of a number by mere calculation. We can also use logarithm table to find the logarithm of a number. The logarithm of a number comprises of two parts. The whole part is the characteristic and the decimal part is the mantissa.

### Positive Characteristic

The whole part or the integral part of a number is the characteristic. The characteristic of the logarithm of any number greater than 1 is positive and is one less than the number of the digits to the left of the decimal point in the given number. If the number is less than one, then the characteristic is negative and is one more than the number of zeros to the right of the decimal point.

**For Example**

| | |
|---|---|
| 111 | 2 |
| 0.1 | – 1 [one more than the number of zeros on the right immediately after the decimal point]. |
| 0.025 | – 2 |
| 0.000010 | – 5 |

| Number | How Many 10s | Base-10 Logarithm | |
|---|---|---|---|
| .. etc.. | | | |
| 1000 | $1 \times 10 \times 10 \times 10$ | $\log_{10}(1000)$ | = 3 |
| 100 | $1 \times 10 \times 10$ | $\log_{10}(100)$ | = 2 |
| 10 | $1 \times 10$ | $\log_{10}(10)$ | = 1 |
| 1 | 1 | $\log_{10}(1)$ | = 0 |
| 0.1 | $1 \div 10$ | $\log_{10}(0.1)$ | = −1 |
| 0.01 | $1 \div 10 \div 10$ | $\log_{10}(0.01)$ | = −2 |
| 0.001 | $1 \div 10 \div 10 \div 10$ | $\log_{10}(0.001)$ | = −3 |
| .. etc.. | | | |

**Negative Characteristic**

The logarithm of a number having 'n' zeros immediately after the decimal is – (n + 1) + the decimal.

**Mantissa**

**The decimal part of the number logarithm of a number is the mantissa**. ***A mantissa is always a positive quantity***. *The negative mantissa should always be converted into a positive one*. For example

**$-5.2592 = -6 + (1 - 0.2592) = \bar{6} + 0.7428$**

**Anti-Logarithms (Antilog)**

**The anti-logarithm of a number is the inverse process of finding the logarithms of the same number**.

If x is the logarithm of a number y with a given base b, then y is the anti-logarithm of (antilog) of x to the base b.

If $\log_b y = x$ then, $y = \text{antilog } x$

*Natural Logarithms and Anti-Logarithms have their base as 2.7183. The Logarithms and Anti-Logarithms with base 10 can be converted into natural Logarithms and Anti-Logarithms by multiplying it by 2.303.*

**Anti-Logarithmic Table**

To find the anti-logarithm of a number we use an anti-logarithmic table. Below are the steps to find the antilog.

- The first step is to separate the characteristic and the mantissa part of the number.
- Use the antilog table to find a corresponding value for the mantissa. The first two digits of the mantissa work as the row number and the third digit is equal to the column number. Note this value.
- The antilog table also includes columns which provide the mean difference. For the same row of the mantissa, the column number in the mean difference is equal to the fourth digit. Note this value.
- Add the values so obtained.
- In the characteristic add one. This value shows the place to put the decimal point. The decimal point is inserted after that many digits from the left.

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | Mean Differences | | | | | | | | |
| 10 | 00000 | 00432 | 00860 | 01284 | 01703 | | | | | | 42 | 85 | 127 | 170 | 212 | 254 | 297 | 339 | 381 |
| | | | | | | 02119 | 02531 | 02938 | 03342 | 03743 | 40 | 81 | 121 | 162 | 202 | 242 | 283 | 323 | 364 |
| 11 | 04139 | 04532 | 04922 | 05308 | 05690 | | | | | | 39 | 77 | 116 | 154 | 193 | 232 | 270 | 309 | 348 |
| | | | | | | 06070 | 06446 | 06819 | 07188 | 07555 | 37 | 74 | 111 | 148 | 185 | 222 | 259 | 296 | 333 |
| 12 | 07918 | 08279 | 08636 | 08991 | 09342 | | | | | | 36 | 71 | 106 | 142 | 177 | 213 | 248 | 284 | 319 |
| | | | | | | 09691 | 10037 | 10380 | 10721 | 11059 | 34 | 68 | 102 | 136 | 170 | 204 | 238 | 272 | 307 |
| 13 | 11394 | 11727 | 12057 | 12385 | 12710 | | | | | | 33 | 66 | 98 | 131 | 164 | 197 | 229 | 262 | 295 |
| | | | | | | 13033 | 13354 | 13672 | 13988 | 14301 | 32 | 63 | 95 | 126 | 158 | 190 | 221 | 253 | 284 |
| 14 | 14613 | 14922 | 15229 | 15534 | 15836 | | | | | | 30 | 61 | 91 | 122 | 152 | 183 | 213 | 244 | 274 |
| | | | | | | 16137 | 16435 | 16732 | 17026 | 17319 | 29 | 59 | 88 | 118 | 147 | 177 | 206 | 236 | 265 |
| 15 | 17609 | 17898 | 18184 | 18469 | 18752 | | | | | | 28 | 57 | 85 | 114 | 142 | 171 | 199 | 228 | 256 |
| | | | | | | 19033 | 19312 | 19590 | 19866 | 20140 | 28 | 55 | 83 | 110 | 138 | 165 | 193 | 221 | 248 |
| 16 | 20412 | 20683 | 20952 | 21219 | 21484 | | | | | | 27 | 53 | 80 | 107 | 134 | 160 | 187 | 214 | 240 |
| | | | | | | 21748 | 22011 | 22272 | 22531 | 22789 | 26 | 52 | 78 | 104 | 130 | 156 | 182 | 208 | 233 |
| 17 | 23045 | 23300 | 23553 | 23805 | 24055 | | | | | | 25 | 51 | 76 | 101 | 126 | 151 | 176 | 201 | 227 |
| | | | | | | 24304 | 24551 | 24797 | 25042 | 25285 | 25 | 49 | 73 | 98 | 122 | 147 | 171 | 196 | 220 |
| 18 | 25527 | 25768 | 26007 | 26245 | 26482 | | | | | | 24 | 48 | 71 | 95 | 119 | 143 | 167 | 190 | 214 |
| | | | | | | 26717 | 26951 | 27184 | 27416 | 27646 | 23 | 46 | 69 | 93 | 116 | 139 | 162 | 185 | 208 |
| 19 | 27875 | 28103 | 28330 | 28556 | 28780 | | | | | | 23 | 45 | 68 | 90 | 113 | 135 | 158 | 180 | 203 |
| | | | | | | 29003 | 29226 | 29447 | 29667 | 29885 | 22 | 44 | 66 | 88 | 110 | 132 | 154 | 176 | 198 |
| 20 | 30103 | 30320 | 30535 | 30750 | 30963 | 31175 | 31387 | 31597 | 31806 | 32015 | 21 | 43 | 64 | 85 | 106 | 127 | 148 | 170 | 190 |
| 21 | 32222 | 32428 | 32634 | 32838 | 33041 | 33244 | 33445 | 33646 | 33846 | 34044 | 20 | 41 | 61 | 81 | 101 | 121 | 141 | 162 | 182 |
| 22 | 34242 | 34439 | 34635 | 34830 | 35025 | 35218 | 35411 | 35603 | 35793 | 35984 | 19 | 39 | 58 | 77 | 97 | 116 | 135 | 154 | 174 |
| 23 | 36173 | 36361 | 36549 | 36736 | 36922 | 37107 | 37291 | 37475 | 37658 | 37840 | 19 | 37 | 56 | 74 | 93 | 111 | 130 | 148 | 167 |
| 24 | 38021 | 38202 | 38382 | 38561 | 38739 | 38917 | 39094 | 39270 | 39445 | 39620 | 18 | 35 | 53 | 71 | 89 | 106 | 124 | 142 | 159 |
| 25 | 39794 | 39967 | 40140 | 40312 | 40483 | 40654 | 40824 | 40993 | 41162 | 41330 | 17 | 34 | 51 | 68 | 85 | 102 | 119 | 136 | 153 |
| 26 | 41497 | 41664 | 41830 | 41996 | 42160 | 42325 | 42488 | 42651 | 42813 | 42975 | 16 | 33 | 49 | 66 | 82 | 98 | 115 | 131 | 148 |
| 27 | 43136 | 43297 | 43457 | 43616 | 43775 | 43933 | 44091 | 44248 | 44404 | 44560 | 16 | 32 | 47 | 63 | 79 | 95 | 111 | 126 | 142 |
| 28 | 44716 | 44871 | 45025 | 45179 | 45332 | 45484 | 45637 | 45788 | 45939 | 46090 | 15 | 30 | 46 | 61 | 76 | 91 | 107 | 122 | 137 |
| 29 | 46240 | 46389 | 46538 | 46687 | 46835 | 46982 | 47129 | 47276 | 47422 | 47567 | 15 | 29 | 44 | 59 | 74 | 88 | 103 | 118 | 132 |
| 30 | 47712 | 47857 | 48001 | 48144 | 48287 | 48430 | 48572 | 48714 | 48855 | 48996 | 14 | 29 | 43 | 57 | 72 | 86 | 100 | 114 | 129 |
| 31 | 49136 | 49276 | 49415 | 49554 | 49693 | 49831 | 49969 | 50106 | 50243 | 50379 | 14 | 28 | 41 | 55 | 69 | 83 | 97 | 110 | 124 |
| 32 | 50515 | 50650 | 50786 | 50920 | 51054 | 51188 | 51322 | 51455 | 51587 | 51720 | 13 | 27 | 40 | 54 | 67 | 80 | 94 | 107 | 121 |
| 33 | 51851 | 51983 | 52114 | 52244 | 52375 | 52504 | 52634 | 52763 | 52892 | 53020 | 13 | 26 | 39 | 52 | 65 | 78 | 91 | 104 | 117 |
| 34 | 53148 | 53275 | 53403 | 53529 | 53656 | 53782 | 53908 | 54033 | 54158 | 54283 | 13 | 25 | 38 | 50 | 63 | 76 | 88 | 101 | 113 |
| 35 | 54407 | 54531 | 54654 | 54777 | 54900 | 55023 | 55145 | 55267 | 55388 | 55509 | 12 | 24 | 37 | 49 | 61 | 73 | 85 | 98 | 110 |
| 36 | 55630 | 55751 | 55871 | 55991 | 56110 | 56229 | 56348 | 56467 | 56585 | 56703 | 12 | 24 | 36 | 48 | 60 | 71 | 83 | 95 | 107 |
| 37 | 56820 | 56937 | 57054 | 57171 | 57287 | 57403 | 57519 | 57634 | 57749 | 57864 | 12 | 23 | 35 | 46 | 58 | 70 | 81 | 93 | 104 |
| 38 | 57978 | 58092 | 58206 | 58320 | 58433 | 58546 | 58659 | 58771 | 58883 | 58995 | 11 | 23 | 34 | 45 | 57 | 68 | 79 | 90 | 102 |
| 39 | 59106 | 59218 | 59329 | 59439 | 59550 | 59660 | 59770 | 59879 | 59988 | 60097 | 11 | 22 | 33 | 44 | 55 | 66 | 77 | 88 | 99 |
| 40 | 60206 | 60314 | 60423 | 60531 | 60638 | 60746 | 60853 | 60959 | 61066 | 61172 | 11 | 21 | 32 | 43 | 54 | 64 | 75 | 86 | 97 |
| 41 | 61278 | 61384 | 61490 | 61595 | 61700 | 61805 | 61909 | 62014 | 62118 | 62221 | 10 | 21 | 31 | 42 | 53 | 63 | 74 | 84 | 95 |
| 42 | 62325 | 62428 | 62531 | 62634 | 62737 | 62839 | 62941 | 63043 | 63144 | 63246 | 10 | 20 | 31 | 41 | 51 | 61 | 71 | 82 | 92 |
| 43 | 63347 | 63448 | 63548 | 63649 | 63749 | 63849 | 63949 | 64048 | 64147 | 64246 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 |
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| 46 | 66276 | 66370 | 66464 | 66558 | 66652 | 66745 | 66839 | 66932 | 67025 | 67117 | 9 | 19 | 28 | 37 | 47 | 56 | 65 | 74 | 84 |
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| 49 | 69020 | 69108 | 69197 | 69285 | 69373 | 69461 | 69548 | 69636 | 69723 | 69810 | 9 | 18 | 26 | 35 | 44 | 53 | 62 | 70 | 79 |



| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | Mean Differences | | | | | | | | |
| 50 | 69897 | 69984 | 70070 | 70157 | 70243 | 70329 | 70415 | 70501 | 70586 | 70672 | 9 | 17 | 26 | 34 | 43 | 52 | 60 | 69 | 77 |
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| 53 | 72428 | 72509 | 72591 | 72673 | 72754 | 72835 | 72916 | 72997 | 73078 | 73159 | 8 | 16 | 24 | 32 | 41 | 49 | 57 | 65 | 73 |
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| 55 | 74036 | 74115 | 74194 | 74273 | 74351 | 74429 | 74507 | 74586 | 74663 | 74741 | 8 | 16 | 23 | 31 | 39 | 47 | 55 | 63 | 70 |
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| 71 | 85126 | 85187 | 85248 | 85309 | 85370 | 85431 | 85491 | 85552 | 85612 | 85673 | 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 55 |
| 72 | 85733 | 85794 | 85854 | 85914 | 85974 | 86034 | 86094 | 86153 | 86213 | 86273 | 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 |
| 73 | 86332 | 86392 | 86451 | 86510 | 86570 | 86629 | 86688 | 86747 | 86806 | 86864 | 6 | 12 | 18 | 24 | 30 | 35 | 41 | 47 | 53 |
| 74 | 86923 | 86982 | 87040 | 87099 | 87157 | 87216 | 87274 | 87332 | 87390 | 87448 | 6 | 12 | 17 | 23 | 29 | 35 | 41 | 47 | 52 |
| 75 | 87506 | 87564 | 87622 | 87679 | 87737 | 87795 | 87852 | 87910 | 87967 | 88024 | 6 | 12 | 17 | 23 | 29 | 35 | 40 | 46 | 52 |
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| 77 | 88649 | 88705 | 88762 | 88818 | 88874 | 88930 | 88986 | 89042 | 89098 | 89154 | 6 | 11 | 17 | 22 | 28 | 34 | 39 | 45 | 50 |
| 78 | 89209 | 89265 | 89321 | 89376 | 89432 | 89487 | 89542 | 89597 | 89653 | 89708 | 6 | 11 | 17 | 22 | 28 | 33 | 39 | 44 | 50 |
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| 86 | 93450 | 93500 | 93551 | 93601 | 93651 | 93702 | 93752 | 93802 | 93852 | 93902 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 |
| 87 | 93952 | 94002 | 94052 | 94101 | 94151 | 94201 | 94250 | 94300 | 94349 | 94399 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 |
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| 89 | 94939 | 94988 | 95036 | 95085 | 95134 | 95182 | 95231 | 95279 | 95328 | 95376 | 5 | 10 | 15 | 19 | 24 | 29 | 34 | 39 | 44 |
| 90 | 95424 | 95472 | 95521 | 95569 | 95617 | 95665 | 95713 | 95761 | 95809 | 95856 | 5 | 10 | 14 | 19 | 24 | 29 | 34 | 38 | 43 |
| 91 | 95904 | 95952 | 95999 | 96047 | 96095 | 96142 | 96190 | 96237 | 96284 | 96332 | 5 | 9 | 14 | 19 | 24 | 28 | 33 | 38 | 42 |
| 92 | 96379 | 96426 | 96473 | 96520 | 96567 | 96614 | 96661 | 96708 | 96755 | 96802 | 5 | 9 | 14 | 19 | 24 | 28 | 33 | 38 | 42 |
| 93 | 96848 | 96895 | 96942 | 96988 | 97035 | 97081 | 97128 | 97174 | 97220 | 97267 | 5 | 9 | 14 | 19 | 23 | 28 | 32 | 37 | 42 |
| 94 | 97313 | 97359 | 97405 | 97451 | 97497 | 97543 | 97589 | 97635 | 97681 | 97727 | 5 | 9 | 14 | 18 | 23 | 28 | 32 | 37 | 42 |
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| 96 | 98227 | 98272 | 98318 | 98363 | 98408 | 98453 | 98498 | 98543 | 98588 | 98632 | 5 | 9 | 14 | 18 | 23 | 27 | 32 | 36 | 41 |
| 97 | 98677 | 98722 | 98767 | 98811 | 98856 | 98900 | 98945 | 98989 | 99034 | 99078 | 4 | 9 | 13 | 18 | 22 | 27 | 31 | 36 | 40 |
| 98 | 99123 | 99167 | 99211 | 99255 | 99300 | 99344 | 99388 | 99432 | 99476 | 99520 | 4 | 9 | 13 | 18 | 22 | 26 | 31 | 35 | 40 |
| 99 | 99564 | 99607 | 99651 | 99695 | 99739 | 99782 | 99826 | 99870 | 99913 | 99957 | 4 | 9 | 13 | 17 | 22 | 26 | 31 | 35 | 39 |

## ANTILOGARITHMS

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Mean Differences | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| .00 | 10000 | 10023 | 10046 | 10069 | 10093 | 10116 | 10139 | 10162 | 10186 | 10209 | 2 | 5 | 7 | 9 | 12 | 14 | 16 | 19 | 21 |
| .01 | 10233 | 10257 | 10280 | 10304 | 10328 | 10351 | 10375 | 10399 | 10423 | 10447 | 2 | 5 | 7 | 10 | 12 | 14 | 17 | 19 | 21 |
| .02 | 10471 | 10495 | 10520 | 10544 | 10568 | 10593 | 10617 | 10641 | 10666 | 10691 | 2 | 5 | 7 | 10 | 12 | 15 | 17 | 20 | 22 |
| .03 | 10715 | 10740 | 10765 | 10789 | 10814 | 10839 | 10864 | 10889 | 10914 | 10940 | 3 | 5 | 8 | 10 | 13 | 15 | 18 | 20 | 23 |
| .04 | 10965 | 10990 | 11015 | 11041 | 11066 | 11092 | 11117 | 11143 | 11169 | 11194 | 3 | 5 | 8 | 10 | 13 | 15 | 18 | 20 | 23 |
| .05 | 11220 | 11246 | 11272 | 11298 | 11324 | 11350 | 11376 | 11402 | 11429 | 11455 | 3 | 5 | 8 | 11 | 13 | 16 | 18 | 21 | 24 |
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| .07 | 11749 | 11776 | 11803 | 11830 | 11858 | 11885 | 11912 | 11940 | 11967 | 11995 | 3 | 5 | 8 | 11 | 14 | 16 | 19 | 22 | 25 |
| .08 | 12023 | 12050 | 12078 | 12106 | 12134 | 12162 | 12190 | 12218 | 12246 | 12274 | 3 | 6 | 8 | 11 | 14 | 17 | 20 | 22 | 25 |
| .09 | 12303 | 12331 | 12359 | 12388 | 12417 | 12445 | 12474 | 12503 | 12531 | 12560 | 3 | 6 | 9 | 11 | 14 | 17 | 20 | 23 | 26 |
| .10 | 12589 | 12618 | 12647 | 12677 | 12706 | 12735 | 12764 | 12794 | 12823 | 12853 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 26 |
| .11 | 12882 | 12912 | 12942 | 12972 | 13002 | 13032 | 13062 | 13092 | 13122 | 13152 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 |
| .12 | 13183 | 13213 | 13243 | 13274 | 13305 | 13335 | 13366 | 13397 | 13428 | 13459 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 25 | 28 |
| .13 | 13490 | 13521 | 13552 | 13583 | 13614 | 13646 | 13677 | 13709 | 13740 | 13772 | 3 | 6 | 9 | 13 | 16 | 19 | 22 | 25 | 28 |
| .14 | 13804 | 13836 | 13868 | 13900 | 13932 | 13964 | 13996 | 14028 | 14060 | 14093 | 3 | 6 | 10 | 13 | 16 | 19 | 22 | 26 | 29 |
| .15 | 14125 | 14158 | 14191 | 14223 | 14256 | 14289 | 14322 | 14355 | 14388 | 14421 | 3 | 7 | 10 | 13 | 16 | 20 | 23 | 26 | 30 |
| .16 | 14454 | 14488 | 14521 | 14555 | 14588 | 14622 | 14655 | 14689 | 14723 | 14757 | 3 | 7 | 10 | 13 | 17 | 20 | 24 | 27 | 30 |
| .17 | 14791 | 14825 | 14859 | 14894 | 14928 | 14962 | 14997 | 15031 | 15066 | 15101 | 3 | 7 | 10 | 14 | 17 | 21 | 24 | 28 | 31 |
| .18 | 15136 | 15171 | 15205 | 15241 | 15276 | 15311 | 15346 | 15382 | 15417 | 15453 | 4 | 7 | 11 | 14 | 18 | 21 | 25 | 28 | 32 |
| .19 | 15488 | 15524 | 15560 | 15596 | 15631 | 15668 | 15704 | 15740 | 15776 | 15812 | 4 | 7 | 11 | 14 | 18 | 22 | 25 | 29 | 32 |
| .20 | 15849 | 15885 | 15922 | 15959 | 15996 | 16032 | 16069 | 16106 | 16144 | 16181 | 4 | 7 | 11 | 15 | 18 | 22 | 26 | 30 | 33 |
| .21 | 16218 | 16255 | 16293 | 16331 | 16368 | 16406 | 16444 | 16482 | 16520 | 16558 | 4 | 8 | 11 | 15 | 19 | 23 | 26 | 30 | 34 |
| .22 | 16596 | 16634 | 16672 | 16711 | 16749 | 16788 | 16827 | 16866 | 16904 | 16943 | 4 | 8 | 12 | 15 | 19 | 23 | 27 | 31 | 35 |
| .23 | 16982 | 17022 | 17061 | 17100 | 17140 | 17179 | 17219 | 17258 | 17298 | 17338 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 |
| .24 | 17378 | 17418 | 17458 | 17498 | 17539 | 17579 | 17620 | 17660 | 17701 | 17742 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 |
| .25 | 17783 | 17824 | 17865 | 17906 | 17947 | 17989 | 18030 | 18072 | 18113 | 18155 | 4 | 8 | 12 | 17 | 21 | 25 | 29 | 33 | 37 |
| .26 | 18197 | 18239 | 18281 | 18323 | 18365 | 18408 | 18450 | 18493 | 18535 | 18578 | 4 | 8 | 13 | 17 | 21 | 25 | 30 | 34 | 38 |
| .27 | 18621 | 18664 | 18707 | 18750 | 18793 | 18836 | 18880 | 18923 | 18967 | 19011 | 4 | 9 | 13 | 17 | 22 | 26 | 30 | 35 | 39 |
| .28 | 19055 | 19099 | 19143 | 19187 | 19231 | 19275 | 19320 | 19364 | 19409 | 19454 | 4 | 9 | 13 | 18 | 22 | 26 | 31 | 35 | 40 |
| .29 | 19498 | 19543 | 19588 | 19634 | 19679 | 19724 | 19770 | 19815 | 19861 | 19907 | 5 | 9 | 14 | 18 | 23 | 27 | 32 | 36 | 41 |
| .30 | 19953 | 19999 | 20045 | 20091 | 20137 | 20184 | 20230 | 20277 | 20324 | 20370 | 5 | 9 | 14 | 19 | 23 | 28 | 32 | 37 | 42 |
| .31 | 20417 | 20464 | 20512 | 20559 | 20606 | 20654 | 20701 | 20749 | 20797 | 20845 | 5 | 10 | 14 | 19 | 24 | 29 | 33 | 38 | 43 |
| .32 | 20893 | 20941 | 20989 | 21038 | 21086 | 21135 | 21184 | 21232 | 21281 | 21330 | 5 | 10 | 15 | 19 | 24 | 29 | 34 | 39 | 44 |
| .33 | 21380 | 21429 | 21478 | 21528 | 21577 | 21627 | 21677 | 21727 | 21777 | 21827 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 |
| .34 | 21878 | 21928 | 21979 | 22029 | 22080 | 22131 | 22182 | 22233 | 22284 | 22336 | 5 | 10 | 15 | 20 | 25 | 31 | 36 | 41 | 46 |
| .35 | 22387 | 22439 | 22491 | 22542 | 22594 | 22646 | 22699 | 22751 | 22803 | 22856 | 5 | 10 | 16 | 21 | 26 | 31 | 37 | 42 | 47 |
| .36 | 22909 | 22961 | 23014 | 23067 | 23121 | 23174 | 23227 | 23281 | 23335 | 23388 | 5 | 11 | 16 | 21 | 27 | 32 | 37 | 43 | 48 |
| .37 | 23442 | 23496 | 23550 | 23605 | 23659 | 23714 | 23768 | 23823 | 23878 | 23933 | 5 | 11 | 16 | 22 | 27 | 33 | 38 | 44 | 49 |
| .38 | 23988 | 24044 | 24099 | 24155 | 24210 | 24266 | 24322 | 24378 | 24434 | 24491 | 6 | 11 | 17 | 22 | 28 | 34 | 39 | 45 | 50 |
| .39 | 24547 | 24604 | 24660 | 24717 | 24774 | 24831 | 24889 | 24946 | 25003 | 25061 | 6 | 11 | 17 | 23 | 29 | 34 | 40 | 46 | 51 |
| .40 | 25119 | 25177 | 25235 | 25293 | 25351 | 25410 | 25468 | 25527 | 25586 | 25645 | 6 | 12 | 18 | 23 | 29 | 35 | 41 | 47 | 53 |
| .41 | 25704 | 25763 | 25823 | 25882 | 25942 | 26002 | 26062 | 26122 | 26182 | 26242 | 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 |
| .42 | 26303 | 26363 | 26424 | 26485 | 26546 | 26607 | 26669 | 26730 | 26792 | 26853 | 6 | 12 | 18 | 24 | 31 | 37 | 43 | 49 | 55 |
| .43 | 26915 | 26977 | 27040 | 27102 | 27164 | 27227 | 27290 | 27353 | 27416 | 27479 | 6 | 13 | 19 | 25 | 31 | 38 | 44 | 50 | 56 |
| .44 | 27542 | 27606 | 27669 | 27733 | 27797 | 27861 | 27925 | 27990 | 28054 | 28119 | 6 | 13 | 19 | 26 | 32 | 39 | 45 | 51 | 58 |
| .45 | 28184 | 28249 | 28314 | 28379 | 28445 | 28510 | 28576 | 28642 | 28708 | 28774 | 7 | 13 | 20 | 26 | 33 | 39 | 46 | 52 | 59 |
| .46 | 28840 | 28907 | 28973 | 29040 | 29107 | 29174 | 29242 | 29309 | 29376 | 29444 | 7 | 13 | 20 | 27 | 34 | 40 | 47 | 54 | 60 |
| .47 | 29512 | 29580 | 29648 | 29717 | 29785 | 29854 | 29923 | 29992 | 30061 | 30130 | 7 | 14 | 21 | 28 | 34 | 41 | 48 | 55 | 62 |
| .48 | 30200 | 30269 | 30339 | 30409 | 30479 | 30549 | 30620 | 30690 | 30761 | 30832 | 7 | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 63 |
| .49 | 30903 | 30974 | 31046 | 31117 | 31189 | 31261 | 31333 | 31405 | 31477 | 31550 | 7 | 14 | 22 | 29 | 36 | 43 | 50 | 58 | 65 |

## ANTILOGARITHMS

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Mean Differences | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| .50 | 31623 | 31696 | 31769 | 31842 | 31916 | 31989 | 32063 | 32137 | 32211 | 32285 | 7 | 15 | 22 | 29 | 37 | 44 | 52 | 59 | 66 |
| .51 | 32359 | 32434 | 32509 | 32584 | 32659 | 32735 | 32810 | 32885 | 32961 | 33037 | 8 | 15 | 23 | 30 | 38 | 45 | 53 | 60 | 68 |
| .52 | 33113 | 33189 | 33266 | 33343 | 33420 | 33497 | 33574 | 33651 | 33729 | 33806 | 8 | 15 | 23 | 31 | 39 | 46 | 54 | 62 | 70 |
| .53 | 33884 | 33963 | 34041 | 34119 | 34198 | 34277 | 34356 | 34435 | 34514 | 34594 | 8 | 16 | 24 | 32 | 40 | 47 | 55 | 63 | 71 |
| .54 | 34674 | 34754 | 34834 | 34914 | 34995 | 35075 | 35156 | 35237 | 35318 | 35400 | 8 | 16 | 24 | 32 | 40 | 48 | 56 | 65 | 73 |
| .55 | 35481 | 35563 | 35645 | 35727 | 35810 | 35892 | 35975 | 36058 | 36141 | 36224 | 8 | 17 | 25 | 33 | 41 | 50 | 58 | 66 | 74 |
| .56 | 36308 | 36392 | 36475 | 36559 | 36644 | 36728 | 36813 | 36898 | 36983 | 37068 | 8 | 17 | 25 | 34 | 42 | 51 | 59 | 68 | 76 |
| .57 | 37154 | 37239 | 37325 | 37411 | 37497 | 37584 | 37670 | 37757 | 37844 | 37931 | 9 | 17 | 26 | 35 | 43 | 52 | 61 | 69 | 78 |
| .58 | 38019 | 38107 | 38194 | 38282 | 38371 | 38459 | 38548 | 38637 | 38726 | 38815 | 9 | 18 | 27 | 35 | 44 | 53 | 62 | 71 | 80 |
| .59 | 38905 | 38994 | 39084 | 39174 | 39264 | 39355 | 39446 | 39537 | 39628 | 39719 | 9 | 18 | 27 | 36 | 45 | 54 | 63 | 72 | 81 |
| .60 | 39811 | 39902 | 39994 | 40087 | 40179 | 40272 | 40365 | 40458 | 40551 | 40644 | 9 | 19 | 28 | 37 | 46 | 56 | 65 | 74 | 83 |
| .61 | 40738 | 40832 | 40926 | 41020 | 41115 | 41210 | 41305 | 41400 | 41495 | 41591 | 9 | 19 | 28 | 38 | 47 | 57 | 66 | 76 | 85 |
| .62 | 41687 | 41783 | 41879 | 41976 | 42073 | 42170 | 42267 | 42364 | 42462 | 42560 | 10 | 19 | 29 | 39 | 49 | 58 | 68 | 78 | 87 |
| .63 | 42658 | 42756 | 42855 | 42954 | 43053 | 43152 | 43251 | 43351 | 43451 | 43551 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 |
| .64 | 43652 | 43752 | 43853 | 43954 | 44055 | 44157 | 44259 | 44361 | 44463 | 44566 | 10 | 20 | 30 | 41 | 51 | 61 | 71 | 81 | 91 |
| .65 | 44668 | 44771 | 44875 | 44978 | 45082 | 45186 | 45290 | 45394 | 45499 | 45604 | 10 | 21 | 31 | 42 | 52 | 62 | 73 | 83 | 94 |
| .66 | 45709 | 45814 | 45920 | 46026 | 46132 | 46238 | 46345 | 46452 | 46559 | 46666 | 11 | 21 | 32 | 43 | 53 | 64 | 75 | 85 | 96 |
| .67 | 46774 | 46881 | 46989 | 47098 | 47206 | 47315 | 47424 | 47534 | 47643 | 47753 | 11 | 22 | 33 | 44 | 54 | 65 | 76 | 87 | 98 |
| .68 | 47863 | 47973 | 48084 | 48195 | 48306 | 48417 | 48529 | 48641 | 48753 | 48865 | 11 | 22 | 33 | 45 | 56 | 67 | 78 | 89 | 100 |
| .69 | 48978 | 49091 | 49204 | 49317 | 49431 | 49545 | 49659 | 49774 | 49888 | 50003 | 11 | 23 | 34 | 46 | 57 | 68 | 80 | 91 | 103 |
| .70 | 50119 | 50234 | 50350 | 50466 | 50582 | 50699 | 50816 | 50933 | 51050 | 51168 | 12 | 23 | 35 | 47 | 58 | 70 | 82 | 93 | 105 |
| .71 | 51286 | 51404 | 51523 | 51642 | 51761 | 51880 | 52000 | 52119 | 52240 | 52360 | 12 | 24 | 36 | 48 | 60 | 72 | 84 | 96 | 108 |
| .72 | 52481 | 52602 | 52723 | 52845 | 52966 | 53088 | 53211 | 53333 | 53456 | 53580 | 12 | 24 | 37 | 49 | 61 | 73 | 85 | 98 | 110 |
| .73 | 53703 | 53827 | 53951 | 54075 | 54200 | 54325 | 54450 | 54576 | 54702 | 54828 | 13 | 25 | 38 | 50 | 63 | 75 | 88 | 100 | 113 |
| .74 | 54954 | 55081 | 55208 | 55336 | 55463 | 55590 | 55719 | 55847 | 55976 | 56105 | 13 | 26 | 38 | 51 | 64 | 77 | 90 | 102 | 115 |
| .75 | 56234 | 56364 | 56494 | 56624 | 56754 | 56885 | 57016 | 57148 | 57280 | 57412 | 13 | 26 | 39 | 52 | 66 | 79 | 92 | 105 | 118 |
| .76 | 57544 | 57677 | 57810 | 57943 | 58076 | 58210 | 58345 | 58479 | 58614 | 58749 | 13 | 27 | 40 | 54 | 67 | 80 | 94 | 107 | 121 |
| .77 | 58884 | 59020 | 59156 | 59293 | 59429 | 59566 | 59704 | 59841 | 59979 | 60117 | 14 | 27 | 41 | 55 | 69 | 82 | 96 | 110 | 123 |
| .78 | 60256 | 60395 | 60534 | 60674 | 60814 | 60954 | 61094 | 61235 | 61376 | 61518 | 14 | 28 | 42 | 56 | 70 | 84 | 98 | 112 | 126 |
| .79 | 61659 | 61802 | 61944 | 62087 | 62230 | 62373 | 62517 | 62661 | 62806 | 62951 | 14 | 29 | 43 | 57 | 72 | 86 | 100 | 115 | 129 |
| .80 | 63096 | 63241 | 63387 | 63533 | 63680 | 63826 | 63973 | 64121 | 64269 | 64417 | 15 | 29 | 44 | 59 | 74 | 88 | 103 | 118 | 132 |
| .81 | 64565 | 64714 | 64863 | 65013 | 65163 | 65313 | 65464 | 65615 | 65766 | 65917 | 15 | 30 | 45 | 60 | 75 | 90 | 105 | 120 | 135 |
| .82 | 66069 | 66222 | 66374 | 66527 | 66681 | 66834 | 66988 | 67143 | 67298 | 67453 | 15 | 31 | 46 | 62 | 77 | 92 | 108 | 123 | 139 |
| .83 | 67608 | 67764 | 67920 | 68077 | 68234 | 68391 | 68549 | 68707 | 68865 | 69024 | 16 | 32 | 47 | 63 | 79 | 95 | 110 | 126 | 142 |
| .84 | 69183 | 69343 | 69503 | 69663 | 69823 | 69984 | 70146 | 70307 | 70469 | 70632 | 16 | 32 | 48 | 64 | 81 | 97 | 113 | 129 | 145 |
| .85 | 70795 | 70958 | 71121 | 71285 | 71450 | 71614 | 71779 | 71945 | 72111 | 72277 | 17 | 33 | 50 | 66 | 83 | 99 | 116 | 132 | 149 |
| .86 | 72444 | 72611 | 72778 | 72946 | 73114 | 73282 | 73451 | 73621 | 73790 | 73961 | 17 | 34 | 51 | 68 | 85 | 101 | 118 | 135 | 152 |
| .87 | 74131 | 74302 | 74473 | 74645 | 74817 | 74989 | 75162 | 75336 | 75509 | 75683 | 17 | 35 | 52 | 69 | 87 | 104 | 121 | 138 | 156 |
| .88 | 75858 | 76033 | 76208 | 76384 | 76560 | 76736 | 76913 | 77090 | 77268 | 77446 | 18 | 35 | 53 | 71 | 89 | 107 | 125 | 142 | 160 |
| .89 | 77625 | 77804 | 77983 | 78163 | 78343 | 78524 | 78705 | 78886 | 79068 | 79250 | 18 | 36 | 54 | 72 | 91 | 109 | 127 | 145 | 163 |
| .90 | 79433 | 79616 | 79799 | 79983 | 80168 | 80353 | 80538 | 80724 | 80910 | 81096 | 19 | 37 | 56 | 74 | 93 | 111 | 130 | 148 | 167 |
| .91 | 81283 | 81470 | 81658 | 81846 | 82035 | 82224 | 82414 | 82604 | 82794 | 82985 | 19 | 38 | 57 | 76 | 95 | 114 | 133 | 152 | 171 |
| .92 | 83176 | 83368 | 83560 | 83753 | 83946 | 84140 | 84333 | 84528 | 84723 | 84918 | 19 | 39 | 58 | 78 | 97 | 116 | 136 | 155 | 175 |
| .93 | 85114 | 85310 | 85507 | 85704 | 85901 | 86099 | 86298 | 86497 | 86696 | 86896 | 20 | 40 | 60 | 79 | 99 | 119 | 139 | 159 | 179 |
| .94 | 87096 | 87297 | 87498 | 87700 | 87902 | 88105 | 88308 | 88512 | 88716 | 88920 | 20 | 41 | 61 | 81 | 102 | 122 | 142 | 162 | 183 |
| .95 | 89125 | 89331 | 89536 | 89743 | 89950 | 90157 | 90365 | 90573 | 90782 | 90991 | 21 | 42 | 62 | 83 | 104 | 125 | 146 | 166 | 187 |
| .96 | 91201 | 91411 | 91622 | 91833 | 92045 | 92257 | 92470 | 92683 | 92897 | 93111 | 21 | 42 | 64 | 85 | 106 | 127 | 149 | 170 | 191 |
| .97 | 93325 | 93541 | 93756 | 93972 | 94189 | 94406 | 94624 | 94842 | 95060 | 95280 | 22 | 43 | 65 | 87 | 109 | 130 | 152 | 174 | 195 |
| .98 | 95499 | 95719 | 95940 | 96161 | 96383 | 96605 | 96828 | 97051 | 97275 | 97499 | 22 | 44 | 67 | 89 | 111 | 133 | 155 | 178 | 200 |
| .99 | 97724 | 97949 | 98175 | 98401 | 98628 | 98855 | 99083 | 99312 | 99541 | 99770 | 23 | 46 | 68 | 91 | 114 | 137 | 160 | 182 | 205 |

## Solved Examples on Logarithms and Anti-Logarithms

**Problem:** Find the value of log 2.8726.

Solution: Here the number of digit to the left of the decimal is 1 so the value of the characteristic will be one less than one i.e., 0. From the log table, the value of 2.8726 is 0.45827. Adding the values of mantissa and the characteristic we find the value of the logarithm. So, log 2.8725 = 0 + 0.45827 = 0.45827.

**Problem:** Calculate the antilog of 3.6552.

Solution: Here we need to find the number whose logarithm is 3.655. From the antilog table, the value corresponding to the row 65 and column 5 is 4508. The mean difference column for the value 2 is 2. Adding these two values, we have 4518 + 2 = 4520. The decimal point is placed in 3 + 1 = 4 digits from the left. So, antilog 3.6552 = 4520.0

## Properties of Logarithms

**Logarithms were quickly adopted by scientists because of various useful properties that simplified long, tedious calculations**.
Expressed in terms of common logarithms, this relationship is given by log $mn$ = log $m$ + log $n$. For example, 100 × 1,000 can be calculated by looking up the logarithms of 100 (2) and 1,000 (3), adding the logarithms together (5), and then finding its antilogarithm (100,000) in the table. Logarithms can also be converted between any positive bases (except that 1 cannot be used as the base since all of its powers are equal to 1).

**Some Base-10 logarithms:**

| Logarithmic laws | | | |
|---|---|---|---|
| Products: | $\log_b mn$ | = | $\log_b m + \log_b n$ |
| Ratios: | $\log_b \frac{m}{n}$ | = | $\log_b m - \log_b n$ |
| Powers: | $\log_b n^p$ | = | $p \log_b n$ |
| Roots: | $\log_b \sqrt[q]{n}$ | = | $\frac{1}{q} \log_b n$ |
| Change of bases: | $\log_b n$ | = | $\log_a n \log_b a$ |

**Theorem 1**

**The logarithm of the product of two numbers say x, and y is equal to the sum of the logarithm of the two numbers**. The base should be the same for both the numbers.

$$\log_b (x\, y) = \log_b x + \log_b y$$

**Proof:** Let $\log_b x = p$ such that $b^p = x$ … (i), and

$\log_b y = q$ such that $b^q = y$ … (ii)

Multiplying (i), and (ii), we have

$b^p \times b^q = x \times y = b^{(p+q)}$ [from the law of indices]

Taking log on both sides, we have,

$\log_b x\, y = p + q = \log_b x + \log_b y$.

**Theorem 2**

**The division of the two numbers is the antilog of the difference of logarithm of the two numbers**. The base should be the same for both the numbers.

$$\log x/y = \log x - \log y$$

**Proof:** Let, $\log_b x = p$ such that $b^p = x$ … (i), and

$\log_b y = q$ such that $b^q = y$ … (ii)

Dividing (i) by (ii), we have

$x/y = b^p/b^q = b^{(p-q)}$ [from the law of indices]

Taking log on both sides, we have,

$\log x/y = p - q = \log x - \log y$

**Theorem 3**

**The logarithm of a number to any other base can be determined by the logarithm of the same number to any given base**. Mathematically, the relation is

$$\log_a x = \log_b x \times \log_a b$$
$$\Rightarrow \log_b x = \log_a x \,/\, \log_a b$$

**Proof:** Let, $\log_a x = p$, $\log_b x = q$, and $\log_a b = r$. From the definition of logarithms, we have

$a^p = x = b^q$, and $a^r = b$.

$b^q = x$ can be written as $(a^r)^q = a^{rq} = x$.

Since, $a^p = b^q = a^{rq} = x$. Comparing the powers, we have

$p = r\,q$

or, $\log_a x = \log_a b \times \log_b x$

or, $\log_b x = \log_a x / \log_a b$.

**Theorem 4**

**The logarithm of a number raised to a power is equal to the index of the power multiplied by the logarithm of the number.** The base is the same in both the conditions.

$$\log_b x^n = n \log_b x.$$

**Proof:** Let $\log_b x = p$ so that $b^p = x$. Raising both sides to power n, we have
$(b^p)^n = x^n \Rightarrow b^{pn} = x^n$

Taking log on both the sides, we have $\log_b x^n = p\,n$

or, $\log_b x^n = n \log_b x$.

- $\log_b (x + y) = \log_b x + \log_b (1 + y/x)$
- $\log_b (x - y) = \log_b x + \log_b (1 - y/x)$

**Logarithms.** The logarithm of a number consists of an integral part called the **characteristic or index**, and decimal part, the **mantissa**.

Referring to the Tables on pages 2-3, 4-5, it will be seen that rows of five figures are placed against each of the numbers from 10 to 99. These five figures form in each case the mantissa of a logarithm; the index, or characteristic, has to be supplied in each case.

**The characteristic of any number greater than unity is positive, and is less by one than the number of figures to the left of the decimal point. The characteristic of a number less than unity is negative and is greater by one than the number of zeros which follow the decimal point.**

| | | | | | | |
|---|---|---|---|---|---|---|
| Characteristic of | 73727 | is 4 ; | characteristic of | | 737·27 | is 2 |
| " " | 73·727 | " 1 | " | " | 7·3727 | " 0 |
| " " | 0·73727 | " $\bar{1}$ | " | " | 0·073727 | " $\bar{2}$ |
| " " | 0·0073727 | " $\bar{3}$ | " | " | 0·00073727 | " $\bar{4}$ |

Negative characteristics are usually designated as *bar* 1, *bar* 2, *bar* 4, etc.

**To find the logarithms of a given number.**

**Ex. 1.** To find log 7·3.

In the column opposite the number 73 is found the mantissa 86332: the characteristic is 0.

Hence, $\log 7{\cdot}3 = 0{\cdot}86332.$

Similarly, $\log 73 = 1{\cdot}86332$ ; $\log 7300 = 3{\cdot}86332.$

**Ex. 2.** Find log 737.

Referring to the tables, find the first two numbers at the extreme left; then, passing along the horizontal line to the number in the vertical column headed by the third figure 7, we obtain the number 86747.

$$\log 737 = 2{\cdot}86747.$$

To obtain the logarithm of a number consisting of four or five figures, it is necessary to use the mean difference columns at the extreme right of the page.

**Ex. 3.** Find log 737·2

$$\begin{aligned} \text{Mantissa of log } 737 &= \underline{0{\cdot}86747} \\ \text{Mean diff. for } 2 &= \quad 12 \\ \therefore \ \log 737{\cdot}2 &= 2{\cdot}86759 \end{aligned}$$

**Ex. 4.** Find log 73·727.

$$\begin{aligned} \text{Mantissa of log } 737 &= 0{\cdot}86747 \,| \\ \text{Mean diff. for } 2 &= \quad 12 \,| \\ \text{" \ " } 7 &= \quad \underline{4 \,|\, 1} \\ \therefore \ \log 73{\cdot}727 &= 1{\cdot}86763 \,| \end{aligned}$$

Similarly, $\log 0{\cdot}0073727 = \bar{3}{\cdot}86763.$

**Ex. 5.** Find log 6425·6

$$\begin{aligned} \text{Mantissa of log } 642 &= 0{\cdot}80754 \,| \\ \text{Mean diff. for } 5 &= \quad 34 \,| \\ \text{" \ " } 6 &= \quad \underline{4 \,|\, 0} \\ &= 0{\cdot}80792 \,| \end{aligned}$$

Hence $\log 6425{\cdot}6 = 3{\cdot}80792$

**Antilogarithms. The number corresponding to a given logarithm is found by using the table antilogarithms.**

**Ex. 6.** Find the number whose log is 1·59584.

| | | |
|---|---|---|
| Antilog. 595 | = | 0·39355 |
| Mean diff. for 8 | = | 72 |
| " " 4 | = | 3 6 |
| | | 0·39431 |

Hence the number whose log is 1·59584 is 39·431. Similarly the number whose log is $\bar{3}$·59584 is 0.0039431.

**Positive and negative characteristics.** It is usual to make the characteristic or index of a logarithm negative; calculations are then most easily carried out. In log $\bar{3}$·59584 only the $\bar{3}$, called bar 3, is negative, the remaining five figures are positive.

**Ex. 7.** Add 3·30535 and $\bar{4}$·54654.

Sum = $\bar{1}$·85189; Sum of indices = $3 + \bar{4} = \bar{1}$.

**Ex. 8.** From $\bar{3}$·74036 subtract $\bar{2}$·87506.

To subtract, change the signs of the latter and add. Thus, 1 carried from the mantissa gives $\bar{4}$, and $\bar{4} + 2 = \bar{2}$.

$\therefore$ Result = $\bar{2}$·86530.

**Multiplication. Add the logarithms of the numbers; the sum is the logarithm of the product.**

**Ex. 9.** Multiply 37·358 by 0·0058343.

Log 37·358 = 1·57238
Log 0·0058343 = $\bar{3}$·76599

Sum = $\bar{1}$·33837 $\therefore$ Product = 0·21796.

**Division. Subtract the logarithm of the division from the logarithm of the dividend; the difference is the logarithm of the quotient.**

**Ex. 10.** Evaluate 0·37358 ÷ 25·687.

Log 0·37358 = $\bar{1}$·57238
Log 25·687 = 1·40972

Difference = $\bar{2}$·16266 $\therefore$ Quotient = 0·014543.

**Ex. 11.** Evaluate $\dfrac{47{\cdot}325 \times 0{\cdot}089712}{69{\cdot}843 \times 3{\cdot}1416}$

Log 47·325 = 1·67509
Log 0·089712 = $\bar{2}$·95285
Log numerator = 0·62794
2·34128
Log result = $\bar{2}$·28666 $\therefore$ Result = 0·019349.

Log 69·843 = 1·84413
Log 3·1416 = 0·49715
Log of denominator = 2·34128

**Involution. Multiply the loarithm of the given number by the index of the power; the product is the logarithm of the required number.**

**Ex. 12.** Find the cube of (*a*) 36·715, (*b*) 0·36715.

(*a*) Log $(36{\cdot}715)^3$ = 1·56485 × 3 = 4·69455.
(*b*) Log $(0{\cdot}36715)^3$ = $\bar{1}$·56485 × 3 = $\bar{2}$·69455.
$\therefore$ $(36{\cdot}715)^3$ = 49494; $(0{\cdot}36715)^3$ = 0·049494.

In (*b*) $3 \times \bar{1} = \bar{3}$, but 1 carried from preceding figure gives $\bar{2}$.