

**Department of Physics
Taki Government College**

STUDY MATERIAL

FOR SEM-2 CORE & SEM-4 GE & DSC

Interference of Light & Inteferometer

Source of the Material: <https://www3.nd.edu/~amoukasi/CBE30361/Useful files/>

Interference of Light Waves

CHAPTER OUTLINE

- 37.1 Conditions for Interference
- 37.2 Young's Double-Slit Experiment
- 37.3 Intensity Distribution of the Double-Slit Interference Pattern
- 37.4 Phasor Addition of Waves
- 37.5 Change of Phase Due to Reflection
- 37.6 Interference in Thin Films
- 37.7 The Michelson Interferometer



▲ The colors in many of a hummingbird's feathers are not due to pigment. The iridescence that makes the brilliant colors that often appear on the throat and belly is due to an interference effect caused by structures in the feathers. The colors will vary with the viewing angle. (RO-MA/Index Stock Imagery)



In the preceding chapter, we used light rays to examine what happens when light passes through a lens or reflects from a mirror. This discussion completed our study of *geometric optics*. Here in Chapter 37 and in the next chapter, we are concerned with *wave optics* or *physical optics*, the study of interference, diffraction, and polarization of light. These phenomena cannot be adequately explained with the ray optics used in Chapters 35 and 36. We now learn how treating light as waves rather than as rays leads to a satisfying description of such phenomena.

37.1 Conditions for Interference

In Chapter 18, we found that the superposition of two mechanical waves can be constructive or destructive. In constructive interference, the amplitude of the resultant wave at a given position or time is greater than that of either individual wave, whereas in destructive interference, the resultant amplitude is less than that of either individual wave. Light waves also interfere with each other. Fundamentally, all interference associated with light waves arises when the electromagnetic fields that constitute the individual waves combine.

If two lightbulbs are placed side by side, no interference effects are observed because the light waves from one bulb are emitted independently of those from the other bulb. The emissions from the two lightbulbs do not maintain a constant phase relationship with each other over time. Light waves from an ordinary source such as a lightbulb undergo random phase changes in time intervals less than a nanosecond. Therefore, the conditions for constructive interference, destructive interference, or some intermediate state are maintained only for such short time intervals. Because the eye cannot follow such rapid changes, no interference effects are observed. Such light sources are said to be **incoherent**.

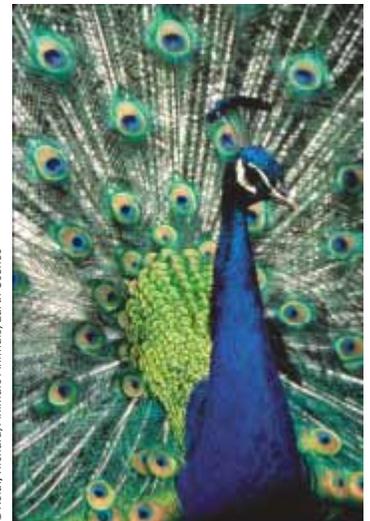
In order to observe interference in light waves, the following conditions must be met:

- The sources must be **coherent**—that is, **they must maintain a constant phase with respect to each other**.
- The sources should be **monochromatic**—that is, of a single wavelength.

As an example, single-frequency sound waves emitted by two side-by-side loudspeakers driven by a single amplifier can interfere with each other because the two speakers are coherent—that is, they respond to the amplifier in the same way at the same time.

37.2 Young's Double-Slit Experiment

A common method for producing two coherent light sources is to use a monochromatic source to illuminate a barrier containing two small openings (usually in the shape of slits). The light emerging from the two slits is coherent because a single



©Kolar, Richard/Animals Animals/Earth Scenes

As with the hummingbird feathers shown in the opening photograph, the bright colors of peacock feathers are also due to interference. In both types of birds, structures in the feathers split and recombine visible light so that interference occurs for certain colors.

Conditions for interference

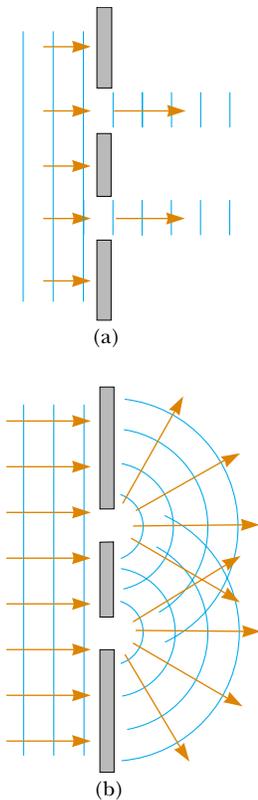


Figure 37.1 (a) If light waves did not spread out after passing through the slits, no interference would occur. (b) The light waves from the two slits overlap as they spread out, filling what we expect to be shadowed regions with light and producing interference fringes on a screen placed to the right of the slits.

▲ PITFALL PREVENTION

37.1 Interference Patterns Are Not Standing Waves

The interference pattern in Figure 37.2b shows bright and dark regions that appear similar to the antinodes and nodes of a standing-wave pattern on a string (Section 18.3). While both patterns depend on the principle of superposition, here are two major differences: (1) waves on a string propagate in only one dimension while the light-wave interference pattern exists in three dimensions; (2) the standing-wave pattern represents no net energy flow, while there is a net energy flow from the slits to the screen in an interference pattern.



Courtesy of Sabina Zigman/Benjamin Cardozo High School

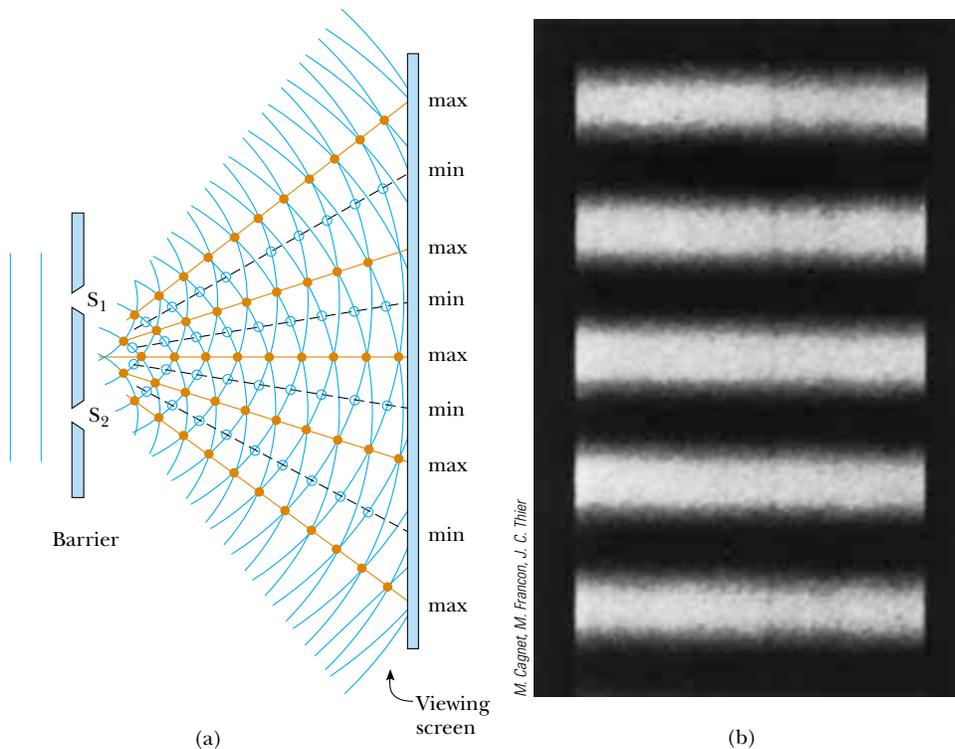
At a beach in Tel Aviv, Israel, plane water waves pass through two openings in a breakwall. Notice the diffraction effect—the waves exit the openings with circular wave fronts, as in Figure 37.1b. Notice also how the beach has been shaped by the circular wave fronts.

source produces the original light beam and the two slits serve only to separate the original beam into two parts (which, after all, is what is done to the sound signal from the side-by-side loudspeakers at the end of the preceding section). Any random change in the light emitted by the source occurs in both beams at the same time, and as a result interference effects can be observed when the light from the two slits arrives at a viewing screen.

If the light traveled only in its original direction after passing through the slits, as shown in Figure 37.1a, the waves would not overlap and no interference pattern would be seen. Instead, as we have discussed in our treatment of Huygens's principle (Section 35.6), the waves spread out from the slits as shown in Figure 37.1b. In other words, the light deviates from a straight-line path and enters the region that would otherwise be shadowed. As noted in Section 35.3, this divergence of light from its initial line of travel is called **diffraction**.

Interference in light waves from two sources was first demonstrated by Thomas Young in 1801. A schematic diagram of the apparatus that Young used is shown in Figure 37.2a. Plane light waves arrive at a barrier that contains two parallel slits S_1 and S_2 . These two slits serve as a pair of coherent light sources because waves emerging from them originate from the same wave front and therefore maintain a constant phase relationship. The light from S_1 and S_2 produces on a viewing screen a visible pattern of bright and dark parallel bands called **fringes** (Fig. 37.2b). When the light from S_1 and that from S_2 both arrive at a point on the screen such that constructive interference occurs at that location, a bright fringe appears. When the light from the two slits combines destructively at any location on the screen, a dark fringe results. Figure 37.3 is a photograph of an interference pattern produced by two coherent vibrating sources in a water tank.

Figure 37.4 shows some of the ways in which two waves can combine at the screen. In Figure 37.4a, the two waves, which leave the two slits in phase, strike the screen at the central point P . Because both waves travel the same distance, they arrive at P in phase. As a result, constructive interference occurs at this location, and a bright fringe is observed. In Figure 37.4b, the two waves also start in phase, but in this case the upper wave has to travel one wavelength farther than the lower



Active Figure 37.2 (a) Schematic diagram of Young's double-slit experiment. Slits S_1 and S_2 behave as coherent sources of light waves that produce an interference pattern on the viewing screen (drawing not to scale). (b) An enlargement of the center of a fringe pattern formed on the viewing screen.

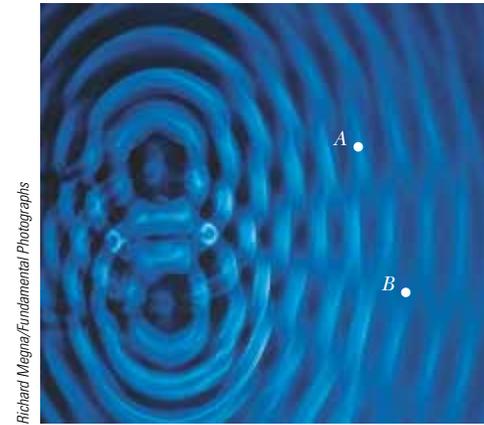


Figure 37.3 An interference pattern involving water waves is produced by two vibrating sources at the water's surface. The pattern is analogous to that observed in Young's double-slit experiment. Note the regions of constructive (A) and destructive (B) interference.

 **At the Active Figures link at <http://www.pse6.com>, you can adjust the slit separation and the wavelength of the light to see the effect on the interference pattern.**

wave to reach point Q . Because the upper wave falls behind the lower one by exactly one wavelength, they still arrive in phase at Q , and so a second bright fringe appears at this location. At point R in Figure 37.4c, however, between points P and Q , the upper wave has fallen half a wavelength behind the lower wave. This means that a trough of the lower wave overlaps a crest of the upper wave; this gives rise to destructive interference at point R . For this reason, a dark fringe is observed at this location.

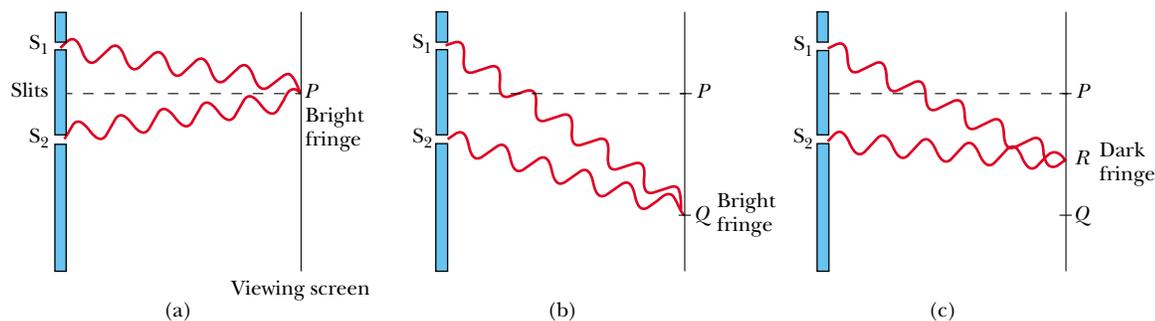


Figure 37.4 (a) Constructive interference occurs at point P when the waves combine. (b) Constructive interference also occurs at point Q . (c) Destructive interference occurs at R when the two waves combine because the upper wave falls half a wavelength behind the lower wave. (All figures not to scale.)

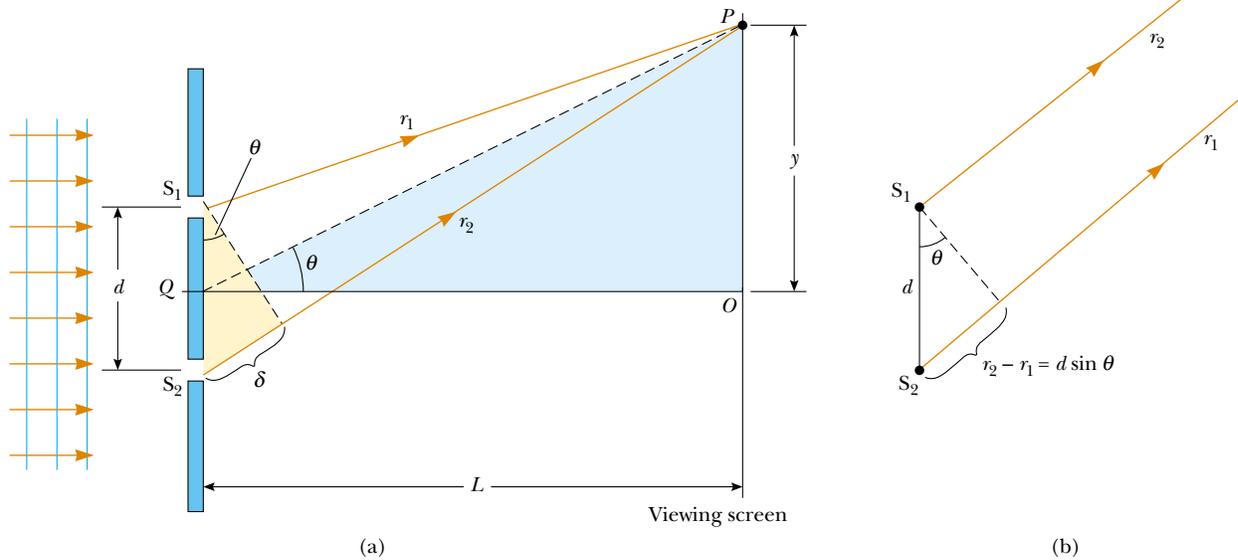


Figure 37.5 (a) Geometric construction for describing Young’s double-slit experiment (not to scale). (b) When we assume that r_1 is parallel to r_2 , the path difference between the two rays is $r_2 - r_1 = d \sin \theta$. For this approximation to be valid, it is essential that $L \gg d$.

We can describe Young’s experiment quantitatively with the help of Figure 37.5. The viewing screen is located a perpendicular distance L from the barrier containing two slits, S_1 and S_2 . These slits are separated by a distance d , and the source is monochromatic. To reach any arbitrary point P in the upper half of the screen, a wave from the lower slit must travel farther than a wave from the upper slit by a distance $d \sin \theta$. This distance is called the **path difference** δ (lowercase Greek delta). If we assume that r_1 and r_2 are parallel, which is approximately true if L is much greater than d , then δ is given by

Path difference

$$\delta = r_2 - r_1 = d \sin \theta \tag{37.1}$$

The value of δ determines whether the two waves are in phase when they arrive at point P . If δ is either zero or some integer multiple of the wavelength, then the two waves are in phase at point P and constructive interference results. Therefore, the condition for bright fringes, or **constructive interference**, at point P is

Conditions for constructive interference

$$\delta = d \sin \theta_{\text{bright}} = m \lambda \quad (m = 0, \pm 1, \pm 2, \dots) \tag{37.2}$$

The number m is called the **order number**. For constructive interference, the order number is the same as the number of wavelengths that represents the path difference between the waves from the two slits. The central bright fringe at $\theta = 0$ is called the *zeroth-order maximum*. The first maximum on either side, where $m = \pm 1$, is called the *first-order maximum*, and so forth.

When δ is an odd multiple of $\lambda/2$, the two waves arriving at point P are 180° out of phase and give rise to destructive interference. Therefore, the condition for dark fringes, or **destructive interference**, at point P is

Conditions for destructive interference

$$d \sin \theta_{\text{dark}} = (m + \frac{1}{2}) \lambda \quad (m = 0, \pm 1, \pm 2, \dots) \tag{37.3}$$

It is useful to obtain expressions for the positions along the screen of the bright and dark fringes measured vertically from O to P . In addition to our assumption that $L \gg d$, we assume $d \gg \lambda$. These can be valid assumptions because in practice L is often on the order of 1 m, d a fraction of a millimeter, and λ a fraction of a micrometer for visible light. Under these conditions, θ is small; thus, we can use the small angle approximation $\sin \theta \approx \tan \theta$. Then, from triangle OPQ in Figure 37.5a,

we see that

$$y = L \tan \theta \approx L \sin \theta \quad (37.4)$$

Solving Equation 37.2 for $\sin \theta$ and substituting the result into Equation 37.4, we see that the positions of the bright fringes measured from O are given by the expression

$$y_{\text{bright}} = \frac{\lambda L}{d} m \quad (m = 0, \pm 1, \pm 2, \dots) \quad (37.5)$$

Using Equations 37.3 and 37.4, we find that the dark fringes are located at

$$y_{\text{dark}} = \frac{\lambda L}{d} \left(m + \frac{1}{2}\right) \quad (m = 0, \pm 1, \pm 2, \dots) \quad (37.6)$$

As we demonstrate in Example 37.1, Young's double-slit experiment provides a method for measuring the wavelength of light. In fact, Young used this technique to do just that. Additionally, his experiment gave the wave model of light a great deal of credibility. It was inconceivable that particles of light coming through the slits could cancel each other in a way that would explain the dark fringes.

Quick Quiz 37.1 If you were to blow smoke into the space between the barrier and the viewing screen of Figure 37.5a, the smoke would show (a) no evidence of interference between the barrier and the screen (b) evidence of interference everywhere between the barrier and the screen.

Quick Quiz 37.2 In a two-slit interference pattern projected on a screen, the fringes are equally spaced on the screen (a) everywhere (b) only for large angles (c) only for small angles.

Quick Quiz 37.3 Which of the following will cause the fringes in a two-slit interference pattern to move farther apart? (a) decreasing the wavelength of the light (b) decreasing the screen distance L (c) decreasing the slit spacing d (d) immersing the entire apparatus in water.

▲ PITFALL PREVENTION

37.2 It May Not Be True That θ Is Small

The approximation $\sin \theta \approx \tan \theta$ is true to three-digit precision only for angles less than about 4° . If you are considering fringes that are far removed from the central fringe, $\tan \theta = y/L$ is still true, but the small-angle approximation may not be valid. In this case, Equations 37.5 and 37.6 cannot be used. These problems can be solved, but the geometry is not as simple.

▲ PITFALL PREVENTION

37.3 It May Not Be True That $L \gg d$

Equations 37.2, 37.3, 37.5, and 37.6 were developed under the assumption that $L \gg d$. This is a very common situation, but you are likely to encounter some situations in which this assumption is not valid. In those cases, the geometric construction will be more complicated, but the general approach outlined here will be similar.

Example 37.1 Measuring the Wavelength of a Light Source

Interactive

A viewing screen is separated from a double-slit source by 1.2 m. The distance between the two slits is 0.030 mm. The second-order bright fringe ($m = 2$) is 4.5 cm from the center line.

(A) Determine the wavelength of the light.

Solution We can use Equation 37.5, with $m = 2$, $y_{\text{bright}} = 4.5 \times 10^{-2}$ m, $L = 1.2$ m, and $d = 3.0 \times 10^{-5}$ m:

$$\begin{aligned} \lambda &= \frac{y_{\text{bright}} d}{mL} = \frac{(4.5 \times 10^{-2} \text{ m})(3.0 \times 10^{-5} \text{ m})}{2(1.2 \text{ m})} \\ &= 5.6 \times 10^{-7} \text{ m} = 560 \text{ nm} \end{aligned}$$

which is in the green range of visible light.

(B) Calculate the distance between adjacent bright fringes.

Solution From Equation 37.5 and the results of part (A), we obtain

$$\begin{aligned} y_{m+1} - y_m &= \frac{\lambda L}{d} (m+1) - \frac{\lambda L}{d} m \\ &= \frac{\lambda L}{d} = \frac{(5.6 \times 10^{-7} \text{ m})(1.2 \text{ m})}{3.0 \times 10^{-5} \text{ m}} \\ &= 2.2 \times 10^{-2} \text{ m} = 2.2 \text{ cm} \end{aligned}$$



Investigate the double-slit interference pattern at the Interactive Worked Example link at <http://www.pse6.com>.

Example 37.2 Separating Double-Slit Fringes of Two Wavelengths

A light source emits visible light of two wavelengths: $\lambda = 430 \text{ nm}$ and $\lambda' = 510 \text{ nm}$. The source is used in a double-slit interference experiment in which $L = 1.50 \text{ m}$ and $d = 0.0250 \text{ mm}$. Find the separation distance between the third-order bright fringes.

Solution Using Equation 37.5, with $m = 3$, we find that the fringe positions corresponding to these two wavelengths are

$$y_{\text{bright}} = \frac{\lambda L}{d} m = 3 \frac{\lambda L}{d} = 3 \frac{(430 \times 10^{-9} \text{ m})(1.50 \text{ m})}{0.0250 \times 10^{-3} \text{ m}} = 7.74 \times 10^{-2} \text{ m}$$

$$y'_{\text{bright}} = \frac{\lambda' L}{d} m = 3 \frac{\lambda' L}{d} = 3 \frac{(510 \times 10^{-9} \text{ m})(1.50 \text{ m})}{0.0250 \times 10^{-3} \text{ m}} = 9.18 \times 10^{-2} \text{ m}$$

Hence, the separation distance between the two fringes is

$$\begin{aligned} \Delta y &= 9.18 \times 10^{-2} \text{ m} - 7.74 \times 10^{-2} \text{ m} \\ &= 1.40 \times 10^{-2} \text{ m} = \mathbf{1.40 \text{ cm}} \end{aligned}$$

What If? What if we examine the entire interference pattern due to the two wavelengths and look for overlapping fringes? Are there any locations on the screen where the bright fringes from the two wavelengths overlap exactly?

Answer We could find such a location by setting the location of any bright fringe due to λ equal to one due to λ' ,

using Equation 37.5:

$$\frac{\lambda L}{d} m = \frac{\lambda' L}{d} m'$$

$$\frac{\lambda}{\lambda'} = \frac{m'}{m}$$

Substituting the wavelengths, we have

$$\frac{m'}{m} = \frac{\lambda}{\lambda'} = \frac{430 \text{ nm}}{510 \text{ nm}} = \frac{43}{51}$$

This might suggest that the 51st bright fringe of the 430-nm light would overlap with the 43rd bright fringe of the 510-nm light. However, if we use Equation 37.5 to find the value of y for these fringes, we find

$$y = 51 \frac{(430 \times 10^{-9} \text{ m})(1.5 \text{ m})}{0.025 \times 10^{-3} \text{ m}} = 1.32 \text{ m} = y'$$

This value of y is comparable to L , so that the small-angle approximation used in Equation 37.4 is *not* valid. This suggests that we should not expect Equation 37.5 to give us the correct result. If you use the exact relationship $y = L \tan \theta$, you can show that the bright fringes do indeed overlap when the same condition, $m'/m = \lambda/\lambda'$, is met (see Problem 44). Thus, the 51st fringe of the 430-nm light does overlap with the 43rd fringe of the 510-nm light, but not at the location of 1.32 m. You are asked to find the correct location as part of Problem 44.

37.3 Intensity Distribution of the Double-Slit Interference Pattern

Note that the edges of the bright fringes in Figure 37.2b are not sharp—there is a gradual change from bright to dark. So far we have discussed the locations of only the centers of the bright and dark fringes on a distant screen. Let us now direct our attention to the intensity of the light at other points between the positions of maximum constructive and destructive interference. In other words, we now calculate the distribution of light intensity associated with the double-slit interference pattern.

Again, suppose that the two slits represent coherent sources of sinusoidal waves such that the two waves from the slits have the same angular frequency ω and a constant phase difference ϕ . The total magnitude of the electric field at point P on the screen in Figure 37.6 is the superposition of the two waves. Assuming that the two waves have the same amplitude E_0 , we can write the magnitude of the electric field at point P due to each wave separately as

$$E_1 = E_0 \sin \omega t \quad \text{and} \quad E_2 = E_0 \sin(\omega t + \phi) \quad (37.7)$$

Although the waves are in phase at the slits, their phase difference ϕ at P depends on the path difference $\delta = r_2 - r_1 = d \sin \theta$. A path difference of λ (for constructive interference) corresponds to a phase difference of 2π rad. A path difference of δ is the same fraction of λ as the phase difference ϕ is of 2π . We can describe this mathematically

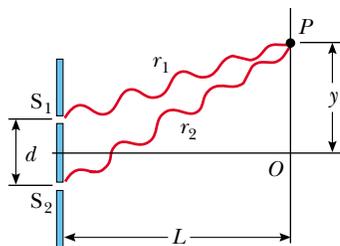


Figure 37.6 Construction for analyzing the double-slit interference pattern. A bright fringe, or intensity maximum, is observed at O .

with the ratio

$$\frac{\delta}{\lambda} = \frac{\phi}{2\pi}$$

which gives us

$$\phi = \frac{2\pi}{\lambda} \delta = \frac{2\pi}{\lambda} d \sin \theta \quad (37.8)$$

Phase difference

This equation tells us precisely how the phase difference ϕ depends on the angle θ in Figure 37.5.

Using the superposition principle and Equation 37.7, we can obtain the magnitude of the resultant electric field at point P :

$$E_P = E_1 + E_2 = E_0[\sin \omega t + \sin(\omega t + \phi)] \quad (37.9)$$

To simplify this expression, we use the trigonometric identity

$$\sin A + \sin B = 2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$$

Taking $A = \omega t + \phi$ and $B = \omega t$, we can write Equation 37.9 in the form

$$E_P = 2E_0 \cos\left(\frac{\phi}{2}\right) \sin\left(\omega t + \frac{\phi}{2}\right) \quad (37.10)$$

This result indicates that the electric field at point P has the same frequency ω as the light at the slits, but that the amplitude of the field is multiplied by the factor $2 \cos(\phi/2)$. To check the consistency of this result, note that if $\phi = 0, 2\pi, 4\pi, \dots$, then the magnitude of the electric field at point P is $2E_0$, corresponding to the condition for maximum constructive interference. These values of ϕ are consistent with Equation 37.2 for constructive interference. Likewise, if $\phi = \pi, 3\pi, 5\pi, \dots$, then the magnitude of the electric field at point P is zero; this is consistent with Equation 37.3 for total destructive interference.

Finally, to obtain an expression for the light intensity at point P , recall from Section 34.3 that *the intensity of a wave is proportional to the square of the resultant electric field magnitude at that point* (Eq. 34.21). Using Equation 37.10, we can therefore express the light intensity at point P as

$$I \propto E_P^2 = 4E_0^2 \cos^2\left(\frac{\phi}{2}\right) \sin^2\left(\omega t + \frac{\phi}{2}\right)$$

Most light-detecting instruments measure time-averaged light intensity, and the time-averaged value of $\sin^2(\omega t + \phi/2)$ over one cycle is $\frac{1}{2}$. (See Figure 33.5.) Therefore, we can write the average light intensity at point P as

$$I = I_{\max} \cos^2\left(\frac{\phi}{2}\right) \quad (37.11)$$

where I_{\max} is the maximum intensity on the screen and the expression represents the time average. Substituting the value for ϕ given by Equation 37.8 into this expression, we find that

$$I = I_{\max} \cos^2\left(\frac{\pi d \sin \theta}{\lambda}\right) \quad (37.12)$$

Alternatively, because $\sin \theta \approx y/L$ for small values of θ in Figure 37.5, we can write Equation 37.12 in the form

$$I \approx I_{\max} \cos^2\left(\frac{\pi d}{\lambda L} y\right) \quad (37.13)$$

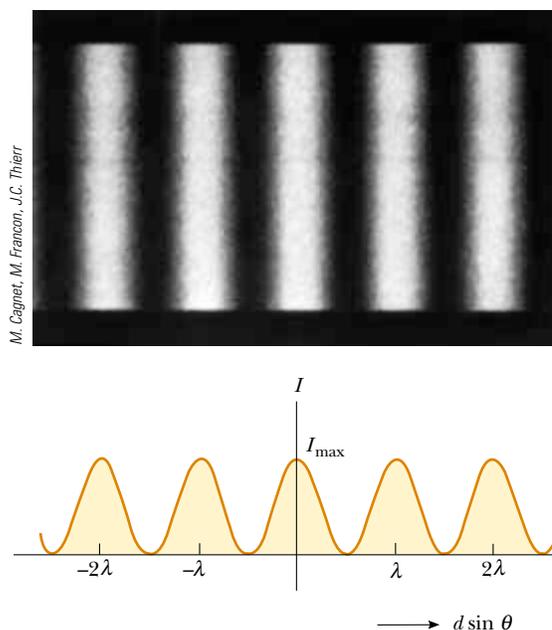


Figure 37.7 Light intensity versus $d \sin \theta$ for a double-slit interference pattern when the screen is far from the two slits ($L \gg d$).

Constructive interference, which produces light intensity maxima, occurs when the quantity $\pi dy/\lambda L$ is an integral multiple of π , corresponding to $y = (\lambda L/d)m$. This is consistent with Equation 37.5.

A plot of light intensity versus $d \sin \theta$ is given in Figure 37.7. The interference pattern consists of equally spaced fringes of equal intensity. Remember, however, that this result is valid only if the slit-to-screen distance L is much greater than the slit separation, and only for small values of θ .

Quick Quiz 37.4 At dark areas in an interference pattern, the light waves have canceled. Thus, there is zero intensity at these regions and, therefore, no energy is arriving. Consequently, when light waves interfere and form an interference pattern, (a) energy conservation is violated because energy disappears in the dark areas (b) energy transferred by the light is transformed to another type of energy in the dark areas (c) the total energy leaving the slits is distributed among light and dark areas and energy is conserved.

37.4 Phasor Addition of Waves

In the preceding section, we combined two waves algebraically to obtain the resultant wave amplitude at some point on a screen. Unfortunately, this analytical procedure becomes cumbersome when we must add several wave amplitudes. Because we shall eventually be interested in combining a large number of waves, we now describe a graphical procedure for this purpose.

Let us again consider a sinusoidal wave whose electric field component is given by

$$E_1 = E_0 \sin \omega t$$

where E_0 is the wave amplitude and ω is the angular frequency. We used phasors in Chapter 33 to analyze AC circuits, and again we find the use of phasors to be valuable

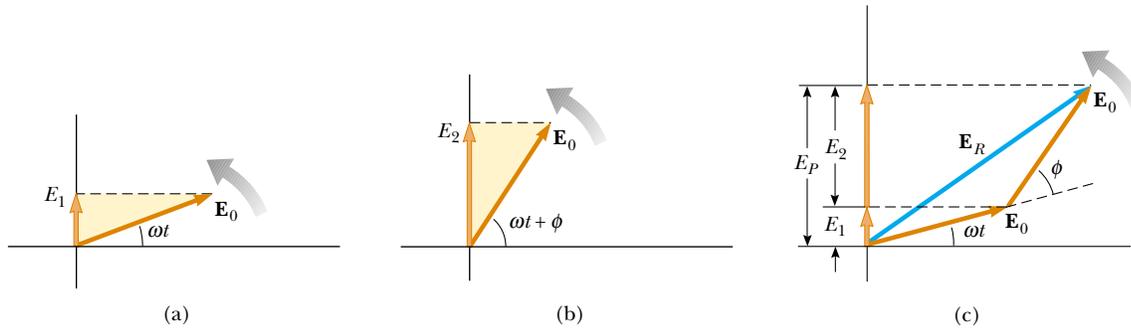


Figure 37.8 (a) Phasor diagram for the wave disturbance $E_1 = E_0 \sin \omega t$. The phasor is a vector of length E_0 rotating counterclockwise. (b) Phasor diagram for the wave $E_2 = E_0 \sin(\omega t + \phi)$. (c) The phasor \mathbf{E}_R represents the combination of the waves in part (a) and (b).

in discussing wave interference. The sinusoidal wave we are discussing can be represented graphically by a phasor of magnitude E_0 rotating about the origin counterclockwise with an angular frequency ω , as in Figure 37.8a. Note that the phasor makes an angle ωt with the horizontal axis. The projection of the phasor on the vertical axis represents E_1 , the magnitude of the wave disturbance at some time t . Hence, as the phasor rotates in a circle about the origin, the projection E_1 oscillates along the vertical axis.

Now consider a second sinusoidal wave whose electric field component is given by

$$E_2 = E_0 \sin(\omega t + \phi)$$

This wave has the same amplitude and frequency as E_1 , but its phase is ϕ with respect to E_1 . The phasor representing E_2 is shown in Figure 37.8b. We can obtain the resultant wave, which is the sum of E_1 and E_2 , graphically by redrawing the phasors as shown in Figure 37.8c, in which the tail of the second phasor is placed at the tip of the first. As with vector addition, the resultant phasor \mathbf{E}_R runs from the tail of the first phasor to the tip of the second. Furthermore, \mathbf{E}_R rotates along with the two individual phasors at the same angular frequency ω . The projection of \mathbf{E}_R along the vertical axis equals the sum of the projections of the two other phasors: $E_P = E_1 + E_2$.

It is convenient to construct the phasors at $t = 0$ as in Figure 37.9. From the geometry of one of the right triangles, we see that

$$\cos \alpha = \frac{E_R/2}{E_0}$$

which gives

$$E_R = 2E_0 \cos \alpha$$

Because the sum of the two opposite interior angles equals the exterior angle ϕ , we see that $\alpha = \phi/2$; thus,

$$E_R = 2E_0 \cos\left(\frac{\phi}{2}\right)$$

Hence, the projection of the phasor \mathbf{E}_R along the vertical axis at any time t is

$$E_P = E_R \sin\left(\omega t + \frac{\phi}{2}\right) = 2E_0 \cos\left(\frac{\phi}{2}\right) \sin\left(\omega t + \frac{\phi}{2}\right)$$

This is consistent with the result obtained algebraically, Equation 37.10. The resultant phasor has an amplitude $2E_0 \cos(\phi/2)$ and makes an angle $\phi/2$ with the first phasor.

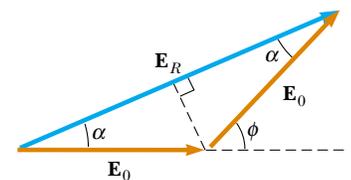


Figure 37.9 A reconstruction of the resultant phasor \mathbf{E}_R . From the geometry, note that $\alpha = \phi/2$.

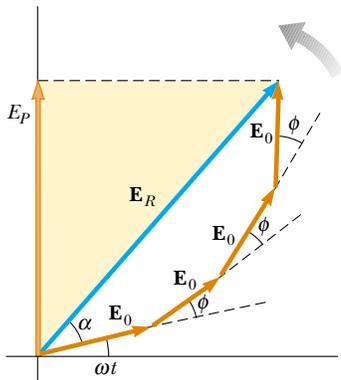


Figure 37.10 The phasor \mathbf{E}_R is the resultant of four phasors of equal amplitude E_0 . The phase of \mathbf{E}_R with respect to the first phasor is α . The projection E_P on the vertical axis represents the combination of the four phasors.

Furthermore, the average light intensity at point P , which varies as E_P^2 , is proportional to $\cos^2(\phi/2)$, as described in Equation 37.11.

We can now describe how to obtain the resultant of several waves that have the same frequency:

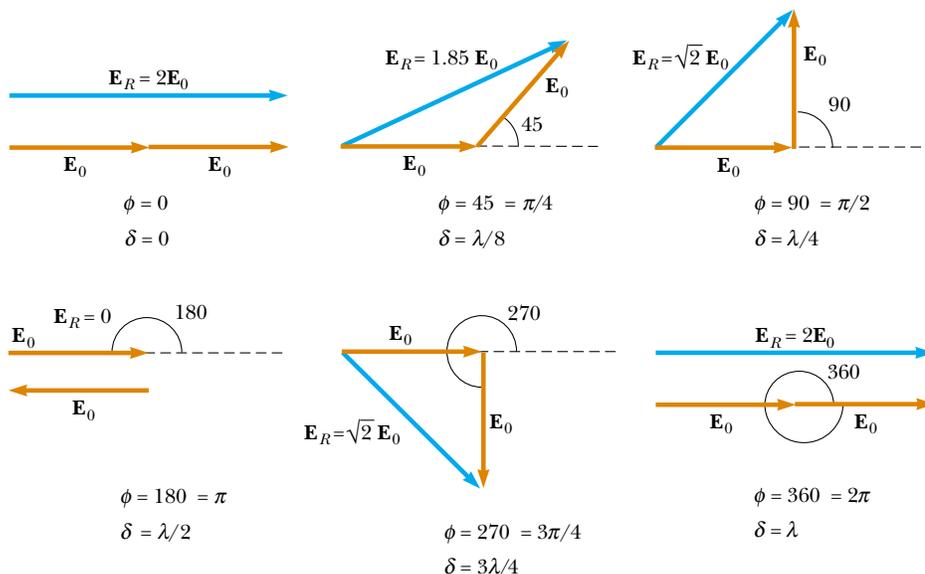
- Represent the waves by phasors, as shown in Figure 37.10, remembering to maintain the proper phase relationship between one phasor and the next.
- The resultant phasor \mathbf{E}_R is the vector sum of the individual phasors. At each instant, the projection of \mathbf{E}_R along the vertical axis represents the time variation of the resultant wave. The phase angle α of the resultant wave is the angle between \mathbf{E}_R and the first phasor. From Figure 37.10, drawn for four phasors, we see that the resultant wave is given by the expression $E_P = E_R \sin(\omega t + \alpha)$.

Phasor Diagrams for Two Coherent Sources

As an example of the phasor method, consider the interference pattern produced by two coherent sources. Figure 37.11 represents the phasor diagrams for various values of the phase difference ϕ and the corresponding values of the path difference δ , which are obtained from Equation 37.8. The light intensity at a point is a maximum when \mathbf{E}_R is a maximum; this occurs at $\phi = 0, 2\pi, 4\pi, \dots$. The light intensity at some point is zero when \mathbf{E}_R is zero; this occurs at $\phi = \pi, 3\pi, 5\pi, \dots$. These results are in complete agreement with the analytical procedure described in the preceding section.

Three-Slit Interference Pattern

Using phasor diagrams, let us analyze the interference pattern caused by three equally spaced slits. We can express the electric field components at a point P on the screen caused by waves from the individual slits as



Active Figure 37.11 Phasor diagrams for a double-slit interference pattern. The resultant phasor \mathbf{E}_R is a maximum when $\phi = 0, 2\pi, 4\pi, \dots$ and is zero when $\phi = \pi, 3\pi, 5\pi, \dots$

 Choose any phase angle at the Active Figures link at <http://www.pse6.com> and see the resultant phasor.

$$E_1 = E_0 \sin \omega t$$

$$E_2 = E_0 \sin(\omega t + \phi)$$

$$E_3 = E_0 \sin(\omega t + 2\phi)$$

where ϕ is the phase difference between waves from adjacent slits. We can obtain the resultant magnitude of the electric field at point P from the phasor diagram in Figure 37.12.

The phasor diagrams for various values of ϕ are shown in Figure 37.13. Note that the resultant magnitude of the electric field at P has a maximum value of $3E_0$, a condition that occurs when $\phi = 0, \pm 2\pi, \pm 4\pi, \dots$. These points are called *primary maxima*. Such primary maxima occur whenever the three phasors are aligned as shown in Figure 37.13a. We also find secondary maxima of amplitude E_0 occurring between the primary maxima at points where $\phi = \pm \pi, \pm 3\pi, \dots$. For these points, the wave from one slit exactly cancels that from another slit (Fig. 37.13d). This means that only light from the third slit contributes to the resultant, which consequently has a total amplitude of E_0 . Total destructive interference occurs whenever the three phasors form a closed triangle, as shown in Figure 37.13c. These points where $E_R = 0$ correspond to $\phi = \pm 2\pi/3, \pm 4\pi/3, \dots$. You should be able to construct other phasor diagrams for values of ϕ greater than π .

Figure 37.14 shows multiple-slit interference patterns for a number of configurations. For three slits, note that the primary maxima are nine times more intense than the secondary maxima as measured by the height of the curve. This is because the intensity varies as E_R^2 . For N slits, the intensity of the primary maxima is N^2 times greater than that due to a single slit. As the number of slits increases, the primary maxima increase in intensity and become narrower, while the secondary maxima decrease in intensity relative to the primary maxima. Figure 37.14 also shows that as the number of slits increases, the number of secondary maxima also increases. In fact, the number of secondary maxima is always $N - 2$ where N is the number of slits. In Section 38.4 (next chapter), we shall investigate the pattern for a very large number of slits in a device called a *diffraction grating*.

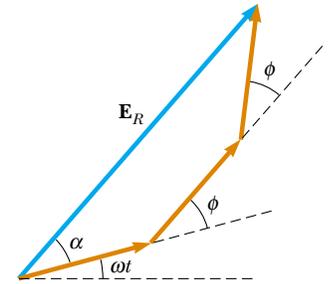
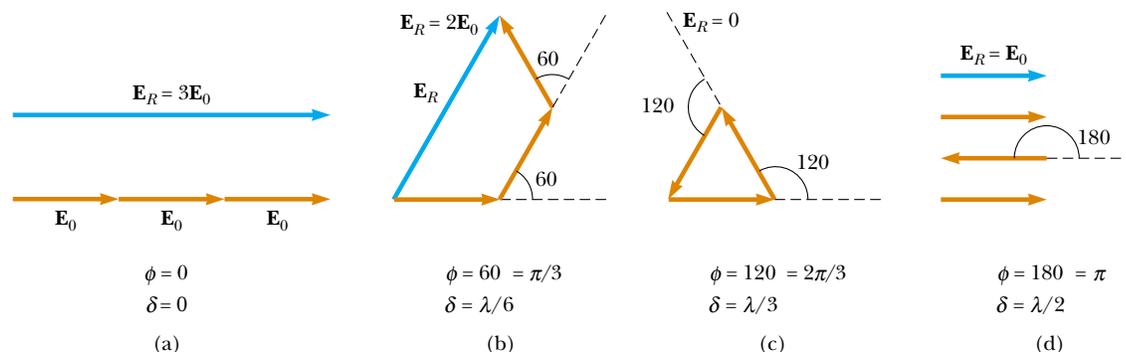


Figure 37.12 Phasor diagram for three equally spaced slits.

Quick Quiz 37.5 Using Figure 37.14 as a model, sketch the interference pattern from six slits.



Active Figure 37.13 Phasor diagrams for three equally spaced slits at various values of ϕ . Note from (a) that there are primary maxima of amplitude $3E_0$ and from (d) that there are secondary maxima of amplitude E_0 .



Choose any phase angle at the Active Figures link at <http://www.pse6.com> and see the resultant phasor.

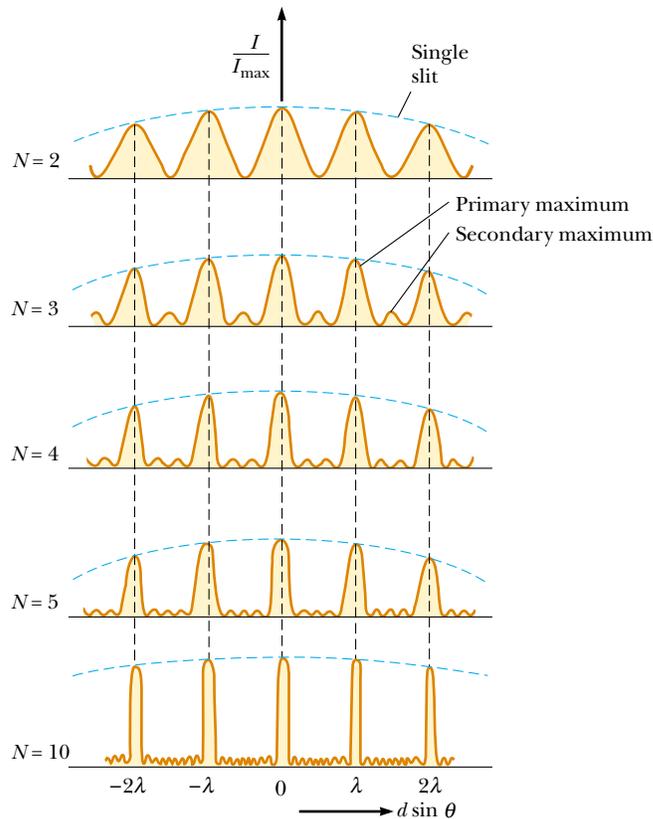


Figure 37.14 Multiple-slit interference patterns. As N , the number of slits, is increased, the primary maxima (the tallest peaks in each graph) become narrower but remain fixed in position and the number of secondary maxima increases. For any value of N , the decrease in intensity in maxima to the left and right of the central maximum, indicated by the blue dashed arcs, is due to *diffraction patterns* from the individual slits, which are discussed in Chapter 38.

37.5 Change of Phase Due to Reflection

Young's method for producing two coherent light sources involves illuminating a pair of slits with a single source. Another simple, yet ingenious, arrangement for producing an interference pattern with a single light source is known as *Lloyd's mirror*¹ (Fig. 37.15). A point light source is placed at point S close to a mirror, and a viewing screen is positioned some distance away and perpendicular to the mirror. Light waves can reach point P on the screen either directly from S to P or by the path involving reflection from the mirror. The reflected ray can be treated as a ray originating from a virtual source at point S' . As a result, we can think of this arrangement as a double-slit source with the distance between points S and S' comparable to length d in Figure 37.5. Hence, at observation points far from the source ($L \gg d$) we expect waves from points S and S' to form an interference pattern just like the one we see from two real coherent sources. An interference pattern is indeed observed. However, the positions of the dark and bright fringes are reversed relative to the pattern created by two real coherent sources (Young's experiment). This can only occur if the coherent sources at points S and S' differ in phase by 180° .

To illustrate this further, consider point P' , the point where the mirror intersects the screen. This point is equidistant from points S and S' . If path difference alone were responsible for the phase difference, we would see a bright fringe at point P' (because the path difference is zero for this point), corresponding to the central bright fringe of

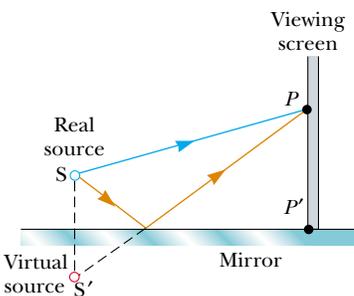


Figure 37.15 Lloyd's mirror. An interference pattern is produced at point P on the screen as a result of the combination of the direct ray (blue) and the reflected ray (brown). The reflected ray undergoes a phase change of 180° .

¹ Developed in 1834 by Humphrey Lloyd (1800–1881), Professor of Natural and Experimental Philosophy, Trinity College, Dublin.

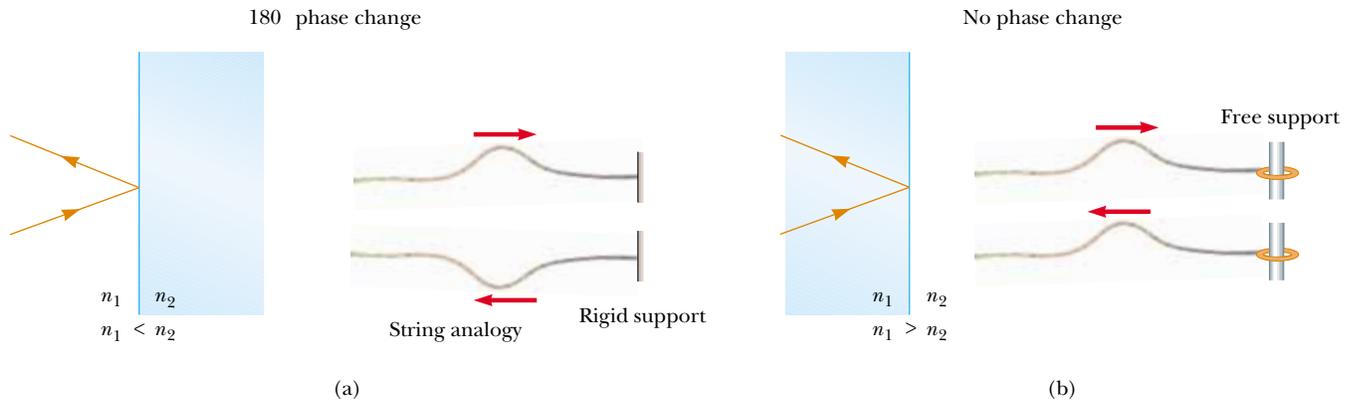


Figure 37.16 (a) For $n_1 < n_2$, a light ray traveling in medium 1 when reflected from the surface of medium 2 undergoes a 180° phase change. The same thing happens with a reflected pulse traveling along a string fixed at one end. (b) For $n_1 > n_2$, a light ray traveling in medium 1 undergoes no phase change when reflected from the surface of medium 2. The same is true of a reflected wave pulse on a string whose supported end is free to move.

the two-slit interference pattern. Instead, we observe a dark fringe at point P' . From this, we conclude that a 180° phase change must be produced by reflection from the mirror. In general, **an electromagnetic wave undergoes a phase change of 180° upon reflection from a medium that has a higher index of refraction than the one in which the wave is traveling.**

It is useful to draw an analogy between reflected light waves and the reflections of a transverse wave pulse on a stretched string (Section 16.4). The reflected pulse on a string undergoes a phase change of 180° when reflected from the boundary of a denser medium, but no phase change occurs when the pulse is reflected from the boundary of a less dense medium. Similarly, an electromagnetic wave undergoes a 180° phase change when reflected from a boundary leading to an optically denser medium (defined as a medium with a higher index of refraction), but no phase change occurs when the wave is reflected from a boundary leading to a less dense medium. These rules, summarized in Figure 37.16, can be deduced from Maxwell's equations, but the treatment is beyond the scope of this text.

37.6 Interference in Thin Films

Interference effects are commonly observed in thin films, such as thin layers of oil on water or the thin surface of a soap bubble. The varied colors observed when white light is incident on such films result from the interference of waves reflected from the two surfaces of the film.

Consider a film of uniform thickness t and index of refraction n , as shown in Figure 37.17. Let us assume that the light rays traveling in air are nearly normal to the two surfaces of the film. To determine whether the reflected rays interfere constructively or destructively, we first note the following facts:

- A wave traveling from a medium of index of refraction n_1 toward a medium of index of refraction n_2 undergoes a 180° phase change upon reflection when $n_2 > n_1$ and undergoes no phase change if $n_2 < n_1$.
- The wavelength of light λ_n in a medium whose index of refraction is n (see Section 35.5) is

$$\lambda_n = \frac{\lambda}{n} \quad (37.14)$$

where λ is the wavelength of the light in free space.

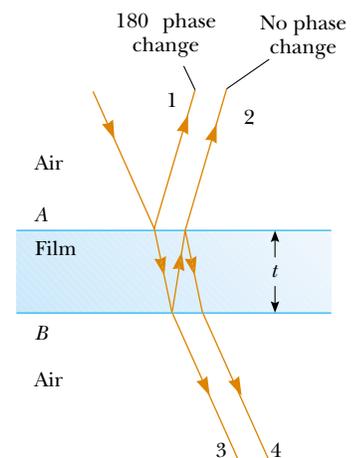


Figure 37.17 Interference in light reflected from a thin film is due to a combination of rays 1 and 2 reflected from the upper and lower surfaces of the film. Rays 3 and 4 lead to interference effects for light transmitted through the film.

Let us apply these rules to the film of Figure 37.17, where $n_{\text{film}} > n_{\text{air}}$. Reflected ray 1, which is reflected from the upper surface (A), undergoes a phase change of 180° with respect to the incident wave. Reflected ray 2, which is reflected from the lower film surface (B), undergoes no phase change because it is reflected from a medium (air) that has a lower index of refraction. Therefore, ray 1 is 180° out of phase with ray 2, which is equivalent to a path difference of $\lambda_n/2$. However, we must also consider that ray 2 travels an extra distance $2t$ before the waves recombine in the air above surface A . (Remember that we are considering light rays that are close to normal to the surface. If the rays are not close to normal, the path difference is larger than $2t$.) If $2t = \lambda_n/2$, then rays 1 and 2 recombine in phase, and the result is constructive interference. In general, the condition for *constructive* interference in thin films is²

$$2t = (m + \frac{1}{2})\lambda_n \quad (m = 0, 1, 2, \dots) \quad (37.15)$$

This condition takes into account two factors: (1) the difference in path length for the two rays (the term $m\lambda_n$) and (2) the 180° phase change upon reflection (the term $\lambda_n/2$). Because $\lambda_n = \lambda/n$, we can write Equation 37.15 as

$$2nt = (m + \frac{1}{2})\lambda \quad (m = 0, 1, 2, \dots) \quad (37.16)$$

If the extra distance $2t$ traveled by ray 2 corresponds to a multiple of λ_n , then the two waves combine out of phase, and the result is destructive interference. The general equation for *destructive* interference in thin films is

$$2nt = m\lambda \quad (m = 0, 1, 2, \dots) \quad (37.17)$$

The foregoing conditions for constructive and destructive interference are valid when the medium above the top surface of the film is the same as the medium below the bottom surface or, if there are different media above and below the film, the index of refraction of both is less than n . If the film is placed between two different media, one with $n < n_{\text{film}}$ and the other with $n > n_{\text{film}}$, then the conditions for constructive and destructive interference are reversed. In this case, either there is a phase change of 180° for both ray 1 reflecting from surface A and ray 2 reflecting from surface B , or there is no phase change for either ray; hence, the net change in relative phase due to the reflections is zero.

Rays 3 and 4 in Figure 37.17 lead to interference effects in the light transmitted through the thin film. The analysis of these effects is similar to that of the reflected light. You are asked to explore the transmitted light in Problems 31, 36, and 37.

Conditions for constructive interference in thin films

Conditions for destructive interference in thin films

▲ PITFALL PREVENTION

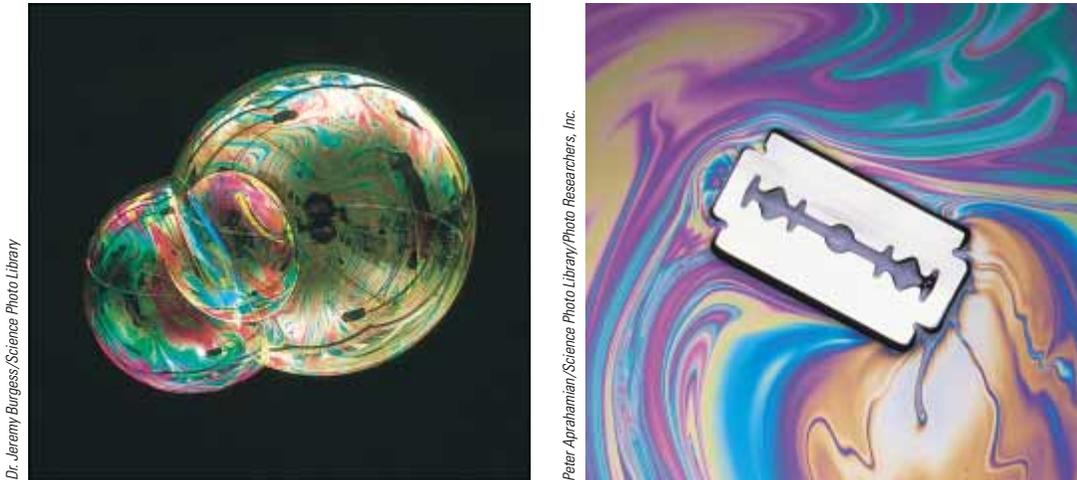
37.4 Be Careful with Thin Films

Be sure to include *both* effects—path length and phase change—when analyzing an interference pattern resulting from a thin film. The possible phase change is a new feature that we did not need to consider for double-slit interference. Also think carefully about the material on either side of the film. You may have situations in which there is a 180° phase change at *both* surfaces or at *neither* surface, if there are different materials on either side of the film.

Quick Quiz 37.6 In a laboratory accident, you spill two liquids onto water, neither of which mixes with the water. They both form thin films on the water surface. When the films become very thin as they spread, you observe that one film becomes bright and the other dark in reflected light. The film that is dark (a) has an index of refraction higher than that of water (b) has an index of refraction lower than that of water (c) has an index of refraction equal to that of water (d) has an index of refraction lower than that of the bright film.

Quick Quiz 37.7 One microscope slide is placed on top of another with their left edges in contact and a human hair under the right edge of the upper slide. As a result, a wedge of air exists between the slides. An interference pattern results when monochromatic light is incident on the wedge. At the left edges of the slides, there is (a) a dark fringe (b) a bright fringe (c) impossible to determine.

² The full interference effect in a thin film requires an analysis of an infinite number of reflections back and forth between the top and bottom surfaces of the film. We focus here only on a single reflection from the bottom of the film, which provides the largest contribution to the interference effect.



(Left) Interference in soap bubbles. The colors are due to interference between light rays reflected from the front and back surfaces of the thin film of soap making up the bubble. The color depends on the thickness of the film, ranging from black where the film is thinnest to magenta where it is thickest. (Right) A thin film of oil floating on water displays interference, as shown by the pattern of colors when white light is incident on the film. Variations in film thickness produce the interesting color pattern. The razor blade gives you an idea of the size of the colored bands.

Newton's Rings

Another method for observing interference in light waves is to place a plano-convex lens on top of a flat glass surface, as shown in Figure 37.18a. With this arrangement, the air film between the glass surfaces varies in thickness from zero at the point of contact to some value t at point P . If the radius of curvature R of the lens is much greater than the distance r , and if the system is viewed from above, a pattern of light and dark rings is observed, as shown in Figure 37.18b. These circular fringes, discovered by Newton, are called **Newton's rings**.

The interference effect is due to the combination of ray 1, reflected from the flat plate, with ray 2, reflected from the curved surface of the lens. Ray 1 undergoes a phase change of 180° upon reflection (because it is reflected from a medium of higher index of refraction), whereas ray 2 undergoes no phase change (because it is reflected from a medium of lower refractive index). Hence, the conditions for constructive and destructive interference are given by Equations 37.16 and 37.17, respectively, with $n = 1$ because the film is air.

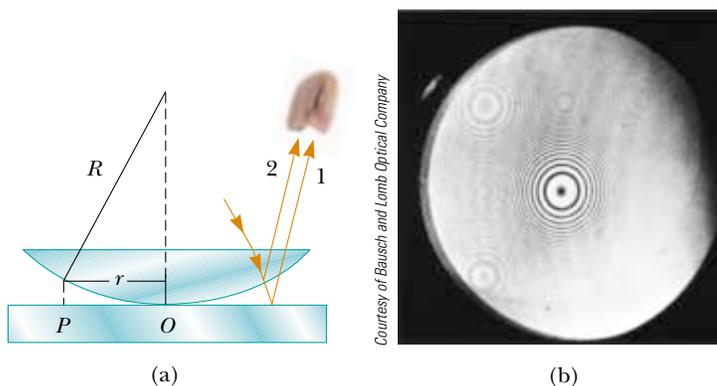


Figure 37.18 (a) The combination of rays reflected from the flat plate and the curved lens surface gives rise to an interference pattern known as Newton's rings. (b) Photograph of Newton's rings.



Figure 37.19 This asymmetrical interference pattern indicates imperfections in the lens of a Newton's-rings apparatus.

The contact point at O is dark, as seen in Figure 37.18b, because there is no path difference and the total phase change is due only to the 180° phase change upon reflection.

Using the geometry shown in Figure 37.18a, we can obtain expressions for the radii of the bright and dark bands in terms of the radius of curvature R and wavelength λ . For example, the dark rings have radii given by the expression $r \approx \pm m\lambda R/n$. The details are left as a problem for you to solve (see Problem 62). We can obtain the wavelength of the light causing the interference pattern by measuring the radii of the rings, provided R is known. Conversely, we can use a known wavelength to obtain R .

One important use of Newton's rings is in the testing of optical lenses. A circular pattern like that pictured in Figure 37.18b is obtained only when the lens is ground to a perfectly symmetric curvature. Variations from such symmetry might produce a pattern like that shown in Figure 37.19. These variations indicate how the lens must be reground and repolished to remove imperfections.

PROBLEM-SOLVING HINTS

Thin-Film Interference

You should keep the following ideas in mind when you work thin-film interference problems:

- Identify the thin film causing the interference.
- The type of interference that occurs is determined by the phase relationship between the portion of the wave reflected at the upper surface of the film and the portion reflected at the lower surface.
- Phase differences between the two portions of the wave have two causes: (1) differences in the distances traveled by the two portions and (2) phase changes that may occur upon reflection.
- When the distance traveled and phase changes upon reflection are both taken into account, the interference is constructive if the equivalent path difference between the two waves is an integral multiple of λ , and it is destructive if the path difference is $\lambda/2$, $3\lambda/2$, $5\lambda/2$, and so forth.

Example 37.3 Interference in a Soap Film

Calculate the minimum thickness of a soap-bubble film that results in constructive interference in the reflected light if the film is illuminated with light whose wavelength in free space is $\lambda = 600$ nm.

Solution The minimum film thickness for constructive interference in the reflected light corresponds to $m = 0$ in Equation 37.16. This gives $2nt = \lambda/2$, or

$$t = \frac{\lambda}{4n} = \frac{600 \text{ nm}}{4(1.33)} = 113 \text{ nm}$$

What If? What if the film is twice as thick? Does this situation produce constructive interference?

Answer Using Equation 37.16, we can solve for the thicknesses at which constructive interference will occur:

$$t = (m + \frac{1}{2}) \frac{\lambda}{2n} = (2m + 1) \frac{\lambda}{4n} \quad (m = 0, 1, 2, \dots)$$

The allowed values of m show that constructive interference will occur for *odd* multiples of the thickness corresponding to $m = 0$, $t = 113$ nm. Thus, constructive interference will *not* occur for a film that is twice as thick.

Example 37.4 Nonreflective Coatings for Solar Cells

Interactive

Solar cells—devices that generate electricity when exposed to sunlight—are often coated with a transparent, thin film of silicon monoxide (SiO , $n = 1.45$) to minimize reflective losses from the surface. Suppose that a silicon solar cell ($n = 3.5$) is coated with a thin film of silicon monoxide for this purpose (Fig. 37.20). Determine the minimum film thickness that

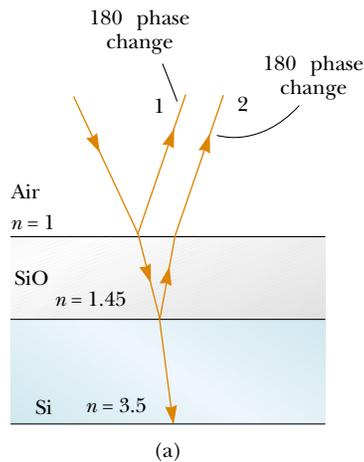


Figure 37.20 (Example 37.4) (a) Reflective losses from a silicon solar cell are minimized by coating the surface of the cell with a thin film of silicon monoxide. (b) The reflected light from a coated camera lens often has a reddish-violet appearance.

produces the least reflection at a wavelength of 550 nm, near the center of the visible spectrum.

Solution Figure 37.20a helps us conceptualize the path of the rays in the SiO film that result in interference in the reflected light. Based on the geometry of the SiO layer, we categorize this as a thin-film interference problem. To analyze the problem, note that the reflected light is a minimum when rays 1 and 2 in Figure 37.20a meet the condition of destructive interference. In this situation, *both* rays undergo a 180° phase change upon reflection—ray 1 from the upper SiO surface and ray 2 from the lower SiO surface. The net change in phase due to reflection is therefore zero, and the condition for a reflection minimum requires a path difference of $\lambda_n/2$, where λ_n is the wavelength of the light in SiO . Hence $2t = \lambda/2n$, where λ is the wavelength in air and n is the index of refraction of SiO . The required thickness is

$$t = \frac{\lambda}{4n} = \frac{550 \text{ nm}}{4(1.45)} = 94.8 \text{ nm}$$

To finalize the problem, we can investigate the losses in typical solar cells. A typical uncoated solar cell has reflective losses as high as 30%; a SiO coating can reduce this value to about 10%. This significant decrease in reflective losses increases the cell's efficiency because less reflection means that more sunlight enters the silicon to create charge carriers in the cell. No coating can ever be made perfectly nonreflecting because the required thickness is wavelength-dependent and the incident light covers a wide range of wavelengths.

Glass lenses used in cameras and other optical instruments are usually coated with a transparent thin film to reduce or eliminate unwanted reflection and enhance the transmission of light through the lenses. The camera lens in Figure 37.20b has several coatings (of different thicknesses) to minimize reflection of light waves having wavelengths near the center of the visible spectrum. As a result, the little light that is reflected by the lens has a greater proportion of the far ends of the spectrum and often appears reddish-violet.

 Investigate the interference for various film properties at the Interactive Worked Example link at <http://www.pse6.com>.

Example 37.5 Interference in a Wedge-Shaped Film

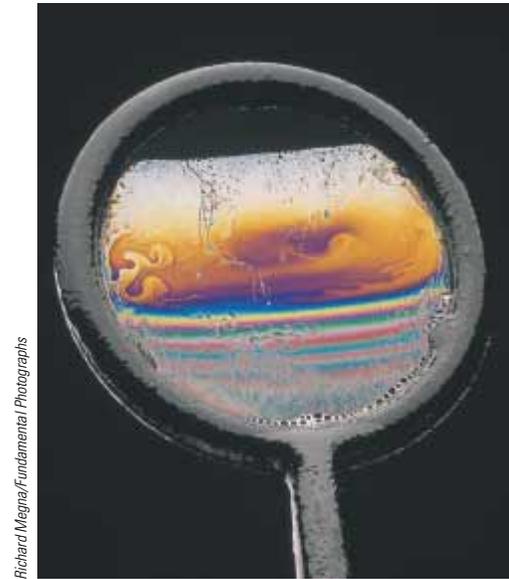
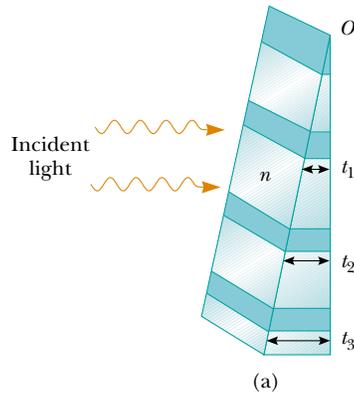
A thin, wedge-shaped film of index of refraction n is illuminated with monochromatic light of wavelength λ , as illustrated in Figure 37.21a. Describe the interference pattern observed for this case.

Solution The interference pattern, because it is created by a thin film of variable thickness surrounded by air, is a series of alternating bright and dark parallel fringes. A dark fringe corresponding to destructive interference appears at point O , the apex, because here the upper reflected ray undergoes a 180° phase change while the lower one undergoes no phase change.

According to Equation 37.17, other dark minima appear when $2nt = m\lambda$; thus, $t_1 = \lambda/2n$, $t_2 = \lambda/n$, $t_3 = 3\lambda/2n$, and so on. Similarly, the bright maxima appear at locations where t satisfies Equation 37.16, $2nt = (m + \frac{1}{2})\lambda$, corresponding to thicknesses of $\lambda/4n$, $3\lambda/4n$, $5\lambda/4n$, and so on.

If white light is used, bands of different colors are observed at different points, corresponding to the different wavelengths of light (see Fig. 37.21b). This is why we see different colors in soap bubbles and other films of varying thickness.

Figure 37.21 (Example 37.5) (a) Interference bands in reflected light can be observed by illuminating a wedge-shaped film with monochromatic light. The darker areas correspond to regions where rays tend to cancel each other because of interference effects. (b) Interference in a vertical film of variable thickness. The top of the film appears darkest where the film is thinnest.



Richard Megna/Fundamental Photographs

(a)

(b)

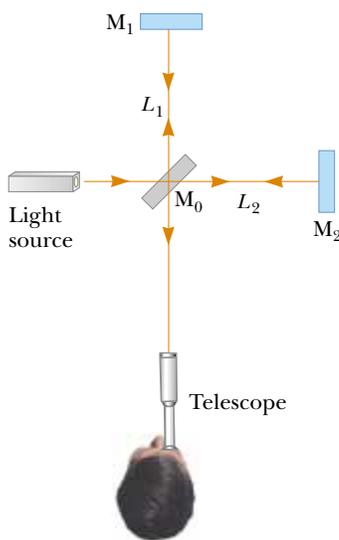
37.7 The Michelson Interferometer

The **interferometer**, invented by the American physicist A. A. Michelson (1852–1931), splits a light beam into two parts and then recombines the parts to form an interference pattern. The device can be used to measure wavelengths or other lengths with great precision because a large and precisely measurable displacement of one of the mirrors is related to an exactly countable number of wavelengths of light.

A schematic diagram of the interferometer is shown in Figure 37.22. A ray of light from a monochromatic source is split into two rays by mirror M_0 , which is inclined at 45° to the incident light beam. Mirror M_0 , called a *beam splitter*, transmits half the light incident on it and reflects the rest. One ray is reflected from M_0 vertically upward toward mirror M_1 , and the second ray is transmitted horizontally through M_0 toward mirror M_2 . Hence, the two rays travel separate paths L_1 and L_2 . After reflecting from M_1 and M_2 , the two rays eventually recombine at M_0 to produce an interference pattern, which can be viewed through a telescope.

The interference condition for the two rays is determined by their path length differences. When the two mirrors are exactly perpendicular to each other, the interference pattern is a target pattern of bright and dark circular fringes, similar to Newton's rings. As M_1 is moved, the fringe pattern collapses or expands, depending on the direction in which M_1 is moved. For example, if a dark circle appears at the center of the target pattern (corresponding to destructive interference) and M_1 is then moved a distance $\lambda/4$ toward M_0 , the path difference changes by $\lambda/2$. What was a dark circle at the center now becomes a bright circle. As M_1 is moved an additional distance $\lambda/4$ toward M_0 , the bright circle becomes a dark circle again. Thus, the fringe pattern shifts by one-half fringe each time M_1 is moved a distance $\lambda/4$. The wavelength of light is then measured by counting the number of fringe shifts for a given displacement of M_1 . If the wavelength is accurately known, mirror displacements can be measured to within a fraction of the wavelength.

We will see an important historical use of the Michelson interferometer in our discussion of relativity in Chapter 39. Modern uses include the following two applications.



Active Figure 37.22 Diagram of the Michelson interferometer. A single ray of light is split into two rays by mirror M_0 , which is called a beam splitter. The path difference between the two rays is varied with the adjustable mirror M_1 . As M_1 is moved, an interference pattern changes in the field of view.

 **At the Active Figures link at <http://www.pse6.com>, move the mirror to see the effect on the interference pattern and use the interferometer to measure the wavelength of light.**

Fourier Transform Infrared Spectroscopy (FTIR)

Spectroscopy is the study of the wavelength distribution of radiation from a sample that can be used to identify the characteristics of atoms or molecules in the sample. Infrared spectroscopy is particularly important to organic chemists in analyzing organic molecules. Traditional spectroscopy involves the use of an optical element, such as a prism (Section 35.7) or a diffraction grating (Section 38.4), which spreads out various wavelengths in a complex optical signal from the sample into different angles. In this way, the various wavelengths of radiation and their intensities in the signal can be determined. These types of devices are limited in their resolution and effectiveness because they must be scanned through the various angular deviations of the radiation.

The technique of *Fourier Transform Infrared Spectroscopy* (FTIR) is used to create a higher-resolution spectrum in a time interval of one second that may have required 30 minutes with a standard spectrometer. In this technique, the radiation from a sample enters a Michelson interferometer. The movable mirror is swept through the zero-path-difference condition and the intensity of radiation at the viewing position is recorded. The result is a complex set of data relating light intensity as a function of mirror position, called an *interferogram*. Because there is a relationship between mirror position and light intensity for a given wavelength, the interferogram contains information about all wavelengths in the signal.

In Section 18.8, we discussed Fourier analysis of a waveform. The waveform is a function that contains information about all of the individual frequency components that make up the waveform.³ Equation 18.16 shows how the waveform is generated from the individual frequency components. Similarly, the interferogram can be analyzed by computer, in a process called a *Fourier transform*, to provide all of the wavelength components. This is the same information generated by traditional spectroscopy, but the resolution of FTIR is much higher.

Laser Interferometer Gravitational-Wave Observatory (LIGO)

Einstein's general theory of relativity (Section 39.10) predicts the existence of *gravitational waves*. These waves propagate from the site of any gravitational disturbance, which could be periodic and predictable, such as the rotation of a double star around a center of mass, or unpredictable, such as the supernova explosion of a massive star.

In Einstein's theory, gravitation is equivalent to a distortion of space. Thus, a gravitational disturbance causes an additional distortion that propagates through space in a manner similar to mechanical or electromagnetic waves. When gravitational waves from a disturbance pass by the Earth, they create a distortion of the local space. The LIGO apparatus is designed to detect this distortion. The apparatus employs a Michelson interferometer that uses laser beams with an effective path length of several kilometers. At the end of an arm of the interferometer, a mirror is mounted on a massive pendulum. When a gravitational wave passes by, the pendulum and the attached mirror move, and the interference pattern due to the laser beams from the two arms changes.

Two sites have been developed in the United States for interferometers in order to allow coincidence studies of gravitational waves. These sites are located in Richland, Washington, and Livingston, Louisiana. Figure 37.23 shows the Washington site. The two arms of the Michelson interferometer are evident in the photograph. Test runs are being performed as of the printing of this book. Cooperation with other gravitational wave detectors, such as VIRGO in Cascina, Italy, will allow detailed studies of gravitational waves.

³ In acoustics, it is common to talk about the components of a complex signal in terms of frequency. In optics, it is more common to identify the components by wavelength.



LIGO Hanford Observatory

Figure 37.23 The Laser Interferometer Gravitational-Wave Observatory (LIGO) near Richland, Washington. Note the two perpendicular arms of the Michelson interferometer.

SUMMARY

 Take a practice test for this chapter by clicking on the Practice Test link at <http://www.pse6.com>.

Interference in light waves occurs whenever two or more waves overlap at a given point. An interference pattern is observed if (1) the sources are coherent and (2) the sources have identical wavelengths.

In Young's double-slit experiment, two slits S_1 and S_2 separated by a distance d are illuminated by a single-wavelength light source. An interference pattern consisting of bright and dark fringes is observed on a viewing screen. The condition for bright fringes (**constructive interference**) is

$$\delta = d \sin \theta_{\text{bright}} = m\lambda \quad (m = 0, \pm 1, \pm 2, \dots) \quad (37.2)$$

The condition for dark fringes (**destructive interference**) is

$$d \sin \theta_{\text{dark}} = (m + \frac{1}{2})\lambda \quad (m = 0, \pm 1, \pm 2, \dots) \quad (37.3)$$

The number m is called the **order number** of the fringe.

The **intensity** at a point in the double-slit interference pattern is

$$I = I_{\text{max}} \cos^2 \left(\frac{\pi d \sin \theta}{\lambda} \right) \quad (37.12)$$

where I_{max} is the maximum intensity on the screen and the expression represents the time average.

A wave traveling from a medium of index of refraction n_1 toward a medium of index of refraction n_2 undergoes a 180° phase change upon reflection when $n_2 > n_1$ and undergoes no phase change when $n_2 < n_1$.

The condition for constructive interference in a film of thickness t and index of refraction n surrounded by air is

$$2nt = (m + \frac{1}{2})\lambda \quad (m = 0, 1, 2, \dots) \quad (37.16)$$

where λ is the wavelength of the light in free space.

Similarly, the condition for destructive interference in a thin film surrounded by air is

$$2nt = m\lambda \quad (m = 0, 1, 2, \dots) \quad (37.17)$$

QUESTIONS

1. What is the necessary condition on the path length difference between two waves that interfere (a) constructively and (b) destructively?
2. Explain why two flashlights held close together do not produce an interference pattern on a distant screen.
3. If Young's double-slit experiment were performed under water, how would the observed interference pattern be affected?
4. In Young's double-slit experiment, why do we use monochromatic light? If white light is used, how would the pattern change?
5. A simple way to observe an interference pattern is to look at a distant light source through a stretched handkerchief or an opened umbrella. Explain how this works.
6. A certain oil film on water appears brightest at the outer regions, where it is thinnest. From this information, what can you say about the index of refraction of oil relative to that of water?
7. As a soap bubble evaporates, it appears black just before it breaks. Explain this phenomenon in terms of the phase changes that occur on reflection from the two surfaces of the soap film.
8. If we are to observe interference in a thin film, why must the film not be very thick (with thickness only on the order of a few wavelengths)?
9. A lens with outer radius of curvature R and index of refraction n rests on a flat glass plate. The combination is illuminated with white light from above and observed from above. Is there a dark spot or a light spot at the center of the lens? What does it mean if the observed rings are noncircular?
10. Why is the lens on a good-quality camera coated with a thin film?
11. Why is it so much easier to perform interference experiments with a laser than with an ordinary light source?
12. Suppose that reflected white light is used to observe a thin transparent coating on glass as the coating material is gradually deposited by evaporation in a vacuum. Describe color changes that might occur during the process of building up the thickness of the coating.
13. In our discussion of thin-film interference, we looked at light *reflecting* from a thin film. **What If?** Consider one light ray, the direct ray, which transmits through the film without reflecting. Consider a second ray, the reflected ray, that transmits through the first surface, reflects from the second, reflects again from the first, and then transmits out into the air, parallel to the direct ray. For normal incidence, how thick must the film be, in terms of the wavelength of light, for the outgoing rays to interfere destructively? Is it the same thickness as for reflected destructive interference?
14. Suppose you are watching television by connection to an antenna rather than a cable system. If an airplane flies near your location, you may notice wavering ghost images in the television picture. What might cause this?

PROBLEMS

- 1, 2, 3 = straightforward, intermediate, challenging □ = full solution available in the *Student Solutions Manual and Study Guide*
 = coached solution with hints available at <http://www.pse6.com>  = computer useful in solving problem
 = paired numerical and symbolic problems

Section 37.1 Conditions for Interference

Section 37.2 Young's Double-Slit Experiment

Note: Problems 8, 9, 10, and 12 in Chapter 18 can be assigned with these sections.

1. A laser beam ($\lambda = 632.8 \text{ nm}$) is incident on two slits 0.200 mm apart. How far apart are the bright interference fringes on a screen 5.00 m away from the double slits?
2. A Young's interference experiment is performed with monochromatic light. The separation between the slits is 0.500 mm , and the interference pattern on a screen 3.30 m away shows the first side maximum 3.40 mm from the center of the pattern. What is the wavelength?
3.  Two radio antennas separated by 300 m as shown in Figure P37.3 simultaneously broadcast identical signals at

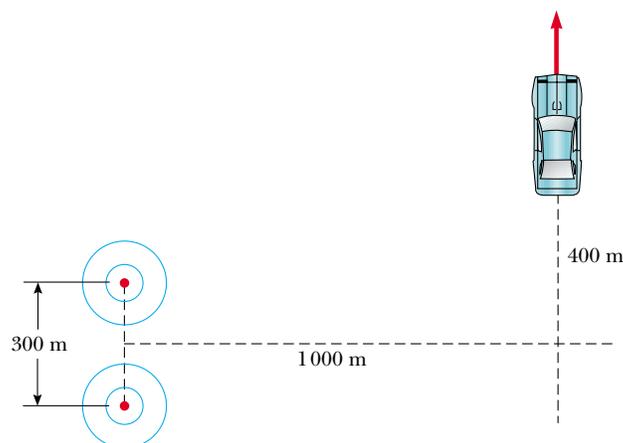


Figure P37.3

the same wavelength. A radio in a car traveling due north receives the signals. (a) If the car is at the position of the second maximum, what is the wavelength of the signals? (b) How much farther must the car travel to encounter the next minimum in reception? (*Note:* Do not use the small-angle approximation in this problem.)

4. In a location where the speed of sound is 354 m/s, a 2000-Hz sound wave impinges on two slits 30.0 cm apart. (a) At what angle is the first maximum located? (b) **What If?** If the sound wave is replaced by 3.00-cm microwaves, what slit separation gives the same angle for the first maximum? (c) **What If?** If the slit separation is $1.00\ \mu\text{m}$, what frequency of light gives the same first maximum angle?

5.  Young's double-slit experiment is performed with 589-nm light and a distance of 2.00 m between the slits and the screen. The tenth interference minimum is observed 7.26 mm from the central maximum. Determine the spacing of the slits.

6. The two speakers of a boom box are 35.0 cm apart. A single oscillator makes the speakers vibrate in phase at a frequency of 2.00 kHz. At what angles, measured from the perpendicular bisector of the line joining the speakers, would a distant observer hear maximum sound intensity? Minimum sound intensity? (Take the speed of sound as 340 m/s.)

7. Two narrow, parallel slits separated by 0.250 mm are illuminated by green light ($\lambda = 546.1\ \text{nm}$). The interference pattern is observed on a screen 1.20 m away from the plane of the slits. Calculate the distance (a) from the central maximum to the first bright region on either side of the central maximum and (b) between the first and second dark bands.

8. Light with wavelength 442 nm passes through a double-slit system that has a slit separation $d = 0.400\ \text{mm}$. Determine how far away a screen must be placed in order that a dark fringe appear directly opposite both slits, with just one bright fringe between them.

9. A riverside warehouse has two open doors as shown in Figure P37.9. Its walls are lined with sound-absorbing material. A boat on the river sounds its horn. To person A the sound is loud and clear. To person B the sound is barely audible. The principal wavelength of the sound waves is 3.00 m. Assuming person B is at the position of the first minimum, determine the distance between the doors, center to center.

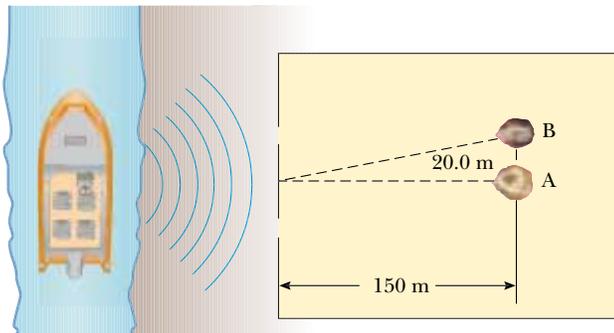


Figure P37.9

10. Two slits are separated by 0.320 mm. A beam of 500-nm light strikes the slits, producing an interference pattern. Determine the number of maxima observed in the angular range $-30.0^\circ < \theta < 30.0^\circ$.

11. Young's double-slit experiment underlies the *Instrument Landing System* used to guide aircraft to safe landings when the visibility is poor. Although real systems are more complicated than the example described here, they operate on the same principles. A pilot is trying to align her plane with a runway, as suggested in Figure P37.11a. Two radio antennas A_1 and A_2 are positioned adjacent to the runway, separated by 40.0 m. The antennas broadcast unmodulated coherent radio waves at 30.0 MHz. (a) Find the wavelength of the waves. The pilot "locks onto" the strong signal radiated along an interference maximum, and steers the plane to keep the received signal strong. (b) **What If?** Suppose instead that the plane is flying along the first side maximum (Fig. P37.11b). How far to the side of the runway centerline will the plane be when it is 2.00 km from the antennas? (c) It is possible to tell the pilot she is on the wrong maximum by sending out two signals from each antenna and equipping the aircraft with a two-channel receiver. The ratio of the two frequencies must not be the ratio of small integers (such as $3/4$). Explain how this two-frequency system would work, and why it would not necessarily work if the frequencies were related by an integer ratio.

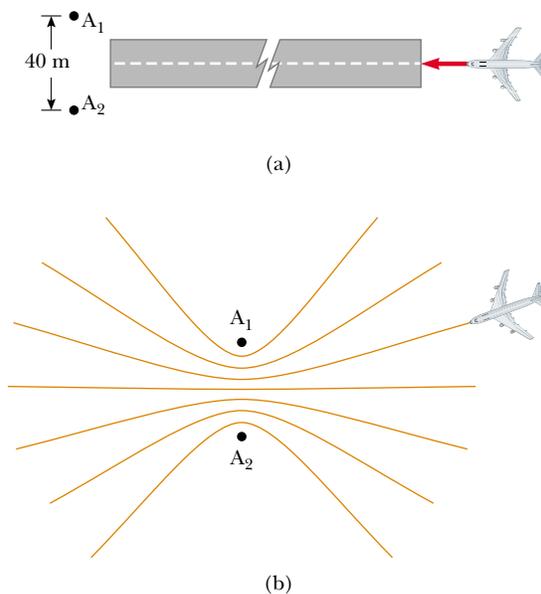


Figure P37.11

12. A student holds a laser that emits light of wavelength 633 nm. The beam passes through a pair of slits separated by 0.300 mm, in a glass plate attached to the front of the laser. The beam then falls perpendicularly on a screen, creating an interference pattern on it. The student begins to walk directly toward the screen at 3.00 m/s. The central maximum on the screen is stationary. Find the speed of the first-order maxima on the screen.

13. In Figure 37.5 let $L = 1.20$ m and $d = 0.120$ mm and assume that the slit system is illuminated with monochromatic 500-nm light. Calculate the phase difference between the two wave fronts arriving at P when (a) $\theta = 0.500^\circ$ and (b) $y = 5.00$ mm. (c) What is the value of θ for which the phase difference is 0.333 rad? (d) What is the value of θ for which the path difference is $\lambda/4$?
14. Coherent light rays of wavelength λ strike a pair of slits separated by distance d at an angle θ_1 as shown in Figure P37.14. Assume an interference maximum is formed at an angle θ_2 a great distance from the slits. Show that $d(\sin\theta_2 - \sin\theta_1) = m\lambda$, where m is an integer.

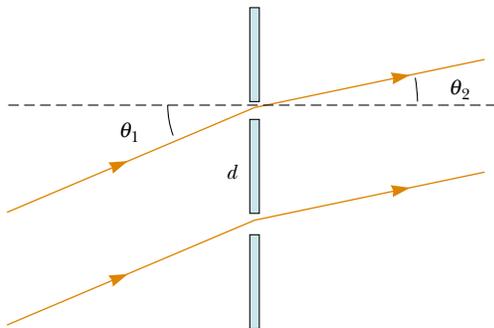


Figure P37.14

15. In a double-slit arrangement of Figure 37.5, $d = 0.150$ mm, $L = 140$ cm, $\lambda = 643$ nm, and $y = 1.80$ cm. (a) What is the path difference δ for the rays from the two slits arriving at P ? (b) Express this path difference in terms of λ . (c) Does P correspond to a maximum, a minimum, or an intermediate condition?

Section 37.3 Intensity Distribution of the Double-Slit Interference Pattern

16. The intensity on the screen at a certain point in a double-slit interference pattern is 64.0% of the maximum value. (a) What minimum phase difference (in radians) between sources produces this result? (b) Express this phase difference as a path difference for 486.1-nm light.
17. In Figure 37.5, let $L = 120$ cm and $d = 0.250$ cm. The slits are illuminated with coherent 600-nm light. Calculate the distance y above the central maximum for which the average intensity on the screen is 75.0% of the maximum.
18. Two slits are separated by 0.180 mm. An interference pattern is formed on a screen 80.0 cm away by 656.3-nm light. Calculate the fraction of the maximum intensity 0.600 cm above the central maximum.
19. Two narrow parallel slits separated by 0.850 mm are illuminated by 600-nm light, and the viewing screen is 2.80 m away from the slits. (a) What is the phase difference between the two interfering waves on a screen at a point 2.50 mm from the central bright fringe? (b) What is the ratio of the intensity at this point to the intensity at the center of a bright fringe?
20. Monochromatic coherent light of amplitude E_0 and angular frequency ω passes through three parallel slits each

separated by a distance d from its neighbor. (a) Show that the time-averaged intensity as a function of the angle θ is

$$I(\theta) = I_{\max} \left[1 + 2 \cos \left(\frac{2\pi d \sin \theta}{\lambda} \right) \right]^2$$

(b) Determine the ratio of the intensities of the primary and secondary maxima.

Section 37.4 Phasor Addition of Waves

Note: Problems 4, 5, and 6 in Chapter 18 can be assigned with this section.

21. Marie Cornu, a physicist at the Polytechnic Institute in Paris, invented phasors in about 1880. This problem helps you to see their utility. Find the amplitude and phase constant of the sum of two waves represented by the expressions

$$E_1 = (12.0 \text{ kN/C}) \sin(15x - 4.5t)$$

$$\text{and } E_2 = (12.0 \text{ kN/C}) \sin(15x - 4.5t + 70^\circ)$$

(a) by using a trigonometric identity (as from Appendix B), and (b) by representing the waves by phasors. (c) Find the amplitude and phase constant of the sum of the three waves represented by

$$E_1 = (12.0 \text{ kN/C}) \sin(15x - 4.5t + 70^\circ),$$

$$E_2 = (15.5 \text{ kN/C}) \sin(15x - 4.5t - 80^\circ),$$

$$\text{and } E_3 = (17.0 \text{ kN/C}) \sin(15x - 4.5t + 160^\circ)$$

22. The electric fields from three coherent sources are described by $E_1 = E_0 \sin \omega t$, $E_2 = E_0 \sin(\omega t + \phi)$, and $E_3 = E_0 \sin(\omega t + 2\phi)$. Let the resultant field be represented by $E_P = E_R \sin(\omega t + \alpha)$. Use phasors to find E_R and α when (a) $\phi = 20.0^\circ$, (b) $\phi = 60.0^\circ$, and (c) $\phi = 120^\circ$. (d) Repeat when $\phi = (3\pi/2)$ rad.

23. Determine the resultant of the two waves given by $E_1 = 6.0 \sin(100 \pi t)$ and $E_2 = 8.0 \sin(100 \pi t + \pi/2)$.

24. Suppose the slit openings in a Young's double-slit experiment have different sizes so that the electric fields and intensities from each slit are different. With $E_1 = E_{01} \sin(\omega t)$ and $E_2 = E_{02} \sin(\omega t + \phi)$, show that the resultant electric field is $E = E_0 \sin(\omega t + \theta)$, where

$$E_0 = \sqrt{E_{01}^2 + E_{02}^2 + 2E_{01}E_{02} \cos \phi}$$

and

$$\sin \theta = \frac{E_{02} \sin \phi}{E_0}$$

25. Use phasors to find the resultant (magnitude and phase angle) of two fields represented by $E_1 = 12 \sin \omega t$ and $E_2 = 18 \sin(\omega t + 60^\circ)$. (Note that in this case the amplitudes of the two fields are unequal.)

26. Two coherent waves are described by

$$E_1 = E_0 \sin \left(\frac{2\pi x_1}{\lambda} - 2\pi f t + \frac{\pi}{6} \right)$$

$$E_2 = E_0 \sin\left(\frac{2\pi x_2}{\lambda} - 2\pi ft + \frac{\pi}{8}\right)$$

Determine the relationship between x_1 and x_2 that produces constructive interference when the two waves are superposed.

27. When illuminated, four equally spaced parallel slits act as multiple coherent sources, each differing in phase from the adjacent one by an angle ϕ . Use a phasor diagram to determine the smallest value of ϕ for which the resultant of the four waves (assumed to be of equal amplitude) is zero.
28. Sketch a phasor diagram to illustrate the resultant of $E_1 = E_{01} \sin \omega t$ and $E_2 = E_{02} \sin(\omega t + \phi)$, where $E_{02} = 1.50E_{01}$ and $\pi/6 \leq \phi \leq \pi/3$. Use the sketch and the law of cosines to show that, for two coherent waves, the resultant intensity can be written in the form $I_R = I_1 + I_2 + 2 I_1 I_2 \cos \phi$.
29. Consider N coherent sources described as follows: $E_1 = E_0 \sin(\omega t + \phi)$, $E_2 = E_0 \sin(\omega t + 2\phi)$, $E_3 = E_0 \sin(\omega t + 3\phi)$, . . . , $E_N = E_0 \sin(\omega t + N\phi)$. Find the minimum value of ϕ for which $E_R = E_1 + E_2 + E_3 + \dots + E_N$ is zero.

Section 37.5 Change of Phase Due to Reflection

Section 37.6 Interference in Thin Films

30. A soap bubble ($n = 1.33$) is floating in air. If the thickness of the bubble wall is 115 nm, what is the wavelength of the light that is most strongly reflected?
31. An oil film ($n = 1.45$) floating on water is illuminated by white light at normal incidence. The film is 280 nm thick. Find (a) the color of the light in the visible spectrum most strongly reflected and (b) the color of the light in the spectrum most strongly transmitted. Explain your reasoning.
32. A thin film of oil ($n = 1.25$) is located on a smooth wet pavement. When viewed perpendicular to the pavement, the film reflects most strongly red light at 640 nm and reflects no blue light at 512 nm. How thick is the oil film?
33. A possible means for making an airplane invisible to radar is to coat the plane with an antireflective polymer. If radar waves have a wavelength of 3.00 cm and the index of refraction of the polymer is $n = 1.50$, how thick would you make the coating?
34. A material having an index of refraction of 1.30 is used as an antireflective coating on a piece of glass ($n = 1.50$). What should be the minimum thickness of this film in order to minimize reflection of 500-nm light?
35. A film of MgF_2 ($n = 1.38$) having thickness 1.00×10^{-5} cm is used to coat a camera lens. Are any wavelengths in the visible spectrum intensified in the reflected light?
36. Astronomers observe the chromosphere of the Sun with a filter that passes the red hydrogen spectral line of wavelength 656.3 nm, called the H_α line. The filter consists of a transparent dielectric of thickness d held between two partially aluminized glass plates. The filter is held at a constant temperature. (a) Find the minimum value of d that produces maximum transmission of perpendicular H_α light, if the dielectric has an index of refraction of 1.378. (b) **What If?** If the temperature of the filter increases

above the normal value, what happens to the transmitted wavelength? (Its index of refraction does not change significantly.) (c) The dielectric will also pass what near-visible wavelength? One of the glass plates is colored red to absorb this light.

37. A beam of 580-nm light passes through two closely spaced glass plates, as shown in Figure P37.37. For what minimum nonzero value of the plate separation d is the transmitted light bright?

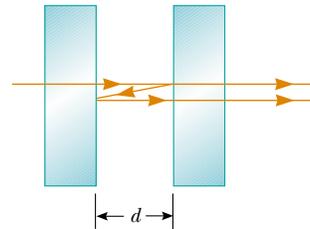


Figure P37.37

38. When a liquid is introduced into the air space between the lens and the plate in a Newton's-rings apparatus, the diameter of the tenth ring changes from 1.50 to 1.31 cm. Find the index of refraction of the liquid.
39. An air wedge is formed between two glass plates separated at one edge by a very fine wire, as shown in Figure P37.39. When the wedge is illuminated from above by 600-nm light and viewed from above, 30 dark fringes are observed. Calculate the radius of the wire.

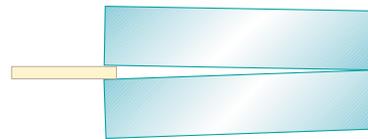


Figure P37.39 Problems 39 and 40.

40. Two glass plates 10.0 cm long are in contact at one end and separated at the other end by a thread 0.050 0 mm in diameter (Fig. P37.39). Light containing the two wavelengths 400 nm and 600 nm is incident perpendicularly and viewed by reflection. At what distance from the contact point is the next dark fringe?

Section 37.7 The Michelson Interferometer

41. Mirror M_1 in Figure 37.22 is displaced a distance ΔL . During this displacement, 250 fringe reversals (formation of successive dark or bright bands) are counted. The light being used has a wavelength of 632.8 nm. Calculate the displacement ΔL .
42. Monochromatic light is beamed into a Michelson interferometer. The movable mirror is displaced 0.382 mm, causing the interferometer pattern to reproduce itself 1 700 times. Determine the wavelength of the light. What color is it?
43. One leg of a Michelson interferometer contains an evacuated cylinder of length L , having glass plates on each end.

A gas is slowly leaked into the cylinder until a pressure of 1 atm is reached. If N bright fringes pass on the screen when light of wavelength λ is used, what is the index of refraction of the gas?

Additional Problems

44. In the **What If?** section of Example 37.2, it was claimed that overlapping fringes in a two-slit interference pattern for two different wavelengths obey the following relation even for large values of the angle θ :

$$\frac{\lambda}{\lambda'} = \frac{m'}{m}$$

(a) Prove this assertion. (b) Using the data in Example 37.2, find the value of y on the screen at which the fringes from the two wavelengths first coincide.

45. One radio transmitter A operating at 60.0 MHz is 10.0 m from another similar transmitter B that is 180° out of phase with A. How far must an observer move from A toward B along the line connecting A and B to reach the nearest point where the two beams are in phase?
46. **Review problem.** This problem extends the result of Problem 12 in Chapter 18. Figure P37.46 shows two adjacent vibrating balls dipping into a tank of water. At distant points they produce an interference pattern of water waves, as shown in Figure 37.3. Let λ represent the wavelength of the ripples. Show that the two sources produce a standing wave along the line segment, of length d , between them. In terms of λ and d , find the number of nodes and the number of antinodes in the standing wave. Find the number of zones of constructive and of destructive interference in the interference pattern far away from the sources. Each line of destructive interference springs from a node in the standing wave and each line of constructive interference springs from an antinode.



Figure P37.46

47. Raise your hand and hold it flat. Think of the space between your index finger and your middle finger as one slit, and think of the space between middle finger and ring finger as a second slit. (a) Consider the interference resulting from sending coherent visible light perpendicularly through this pair of openings. Compute an order-of-magnitude estimate for the angle between adjacent zones

of constructive interference. (b) To make the angles in the interference pattern easy to measure with a plastic protractor, you should use an electromagnetic wave with frequency of what order of magnitude? How is this wave classified on the electromagnetic spectrum?

48. In a Young's double-slit experiment using light of wavelength λ , a thin piece of Plexiglas having index of refraction n covers one of the slits. If the center point on the screen is a dark spot instead of a bright spot, what is the minimum thickness of the Plexiglas?
49. **Review problem.** A flat piece of glass is held stationary and horizontal above the flat top end of a 10.0-cm-long vertical metal rod that has its lower end rigidly fixed. The thin film of air between the rod and glass is observed to be bright by reflected light when it is illuminated by light of wavelength 500 nm. As the temperature is slowly increased by 25.0°C , the film changes from bright to dark and back to bright 200 times. What is the coefficient of linear expansion of the metal?
50. A certain crude oil has an index of refraction of 1.25. A ship dumps 1.00 m^3 of this oil into the ocean, and the oil spreads into a thin uniform slick. If the film produces a first-order maximum of light of wavelength 500 nm normally incident on it, how much surface area of the ocean does the oil slick cover? Assume that the index of refraction of the ocean water is 1.34.
51. Astronomers observe a 60.0-MHz radio source both directly and by reflection from the sea. If the receiving dish is 20.0 m above sea level, what is the angle of the radio source above the horizon at first maximum?
52. Interference effects are produced at point P on a screen as a result of direct rays from a 500-nm source and reflected rays from the mirror, as shown in Figure P37.52. Assume the source is 100 m to the left of the screen and 1.00 cm above the mirror. Find the distance y to the first dark band above the mirror.

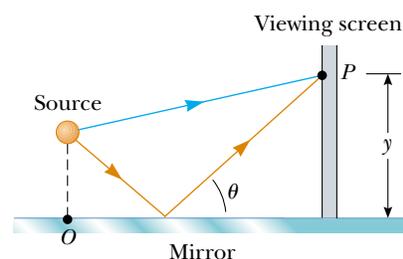


Figure P37.52

53. The waves from a radio station can reach a home receiver by two paths. One is a straight-line path from transmitter to home, a distance of 30.0 km. The second path is by reflection from the ionosphere (a layer of ionized air molecules high in the atmosphere). Assume this reflection takes place at a point midway between receiver and transmitter and that the wavelength broadcast by the radio station is 350 m. Find the minimum height of the ionospheric layer that could produce destructive interference between the direct and reflected beams. (Assume that no phase change occurs on reflection.)

54. Many cells are transparent and colorless. Structures of great interest in biology and medicine can be practically invisible to ordinary microscopy. An *interference microscope* reveals a difference in index of refraction as a shift in interference fringes, to indicate the size and shape of cell structures. The idea is exemplified in the following problem: An air wedge is formed between two glass plates in contact along one edge and slightly separated at the opposite edge. When the plates are illuminated with monochromatic light from above, the reflected light has 85 dark fringes. Calculate the number of dark fringes that appear if water ($n = 1.33$) replaces the air between the plates.

55. Measurements are made of the intensity distribution in a Young's interference pattern (see Fig. 37.7). At a particular value of y , it is found that $I/I_{\max} = 0.810$ when 600-nm light is used. What wavelength of light should be used to reduce the relative intensity at the same location to 64.0% of the maximum intensity?

56. Our discussion of the techniques for determining constructive and destructive interference by reflection from a thin film in air has been confined to rays striking the film at nearly normal incidence. **What If?** Assume that a ray is incident at an angle of 30.0° (relative to the normal) on a film with index of refraction 1.38. Calculate the minimum thickness for constructive interference of sodium light with a wavelength of 590 nm.

57. The condition for constructive interference by reflection from a thin film in air as developed in Section 37.6 assumes nearly normal incidence. **What If?** Show that if the light is incident on the film at a nonzero angle ϕ_1 (relative to the normal), then the condition for constructive interference is $2nt \cos \theta_2 = (m + \frac{1}{2})\lambda$, where θ_2 is the angle of refraction.

58. (a) Both sides of a uniform film that has index of refraction n and thickness d are in contact with air. For normal incidence of light, an intensity minimum is observed in the reflected light at λ_2 and an intensity maximum is observed at λ_1 , where $\lambda_1 > \lambda_2$. Assuming that no intensity minima are observed between λ_1 and λ_2 , show that the integer m in Equations 37.16 and 37.17 is given by $m = \lambda_1/2(\lambda_1 - \lambda_2)$. (b) Determine the thickness of the film, assuming $n = 1.40$, $\lambda_1 = 500$ nm, and $\lambda_2 = 370$ nm.

59. Figure P37.59 shows a radio-wave transmitter and a receiver separated by a distance d and both a distance h above the ground. The receiver can receive signals both directly from

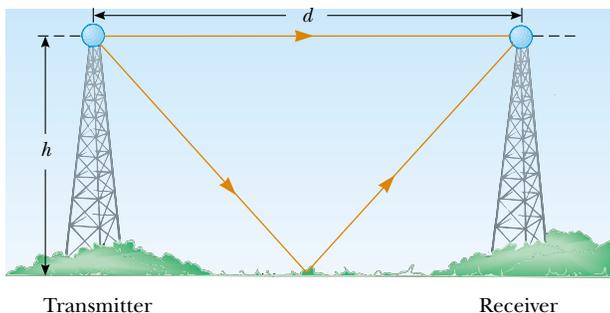


Figure P37.59

the transmitter and indirectly from signals that reflect from the ground. Assume that the ground is level between the transmitter and receiver and that a 180° phase shift occurs upon reflection. Determine the longest wavelengths that interfere (a) constructively and (b) destructively.

60. A piece of transparent material having an index of refraction n is cut into the shape of a wedge as shown in Figure P37.60. The angle of the wedge is small. Monochromatic light of wavelength λ is normally incident from above, and viewed from above. Let h represent the height of the wedge and ℓ its width. Show that bright fringes occur at the positions $x = \lambda\ell(m + \frac{1}{2})/2hn$ and dark fringes occur at the positions $x = \lambda\ell m/2hn$, where $m = 0, 1, 2, \dots$ and x is measured as shown.

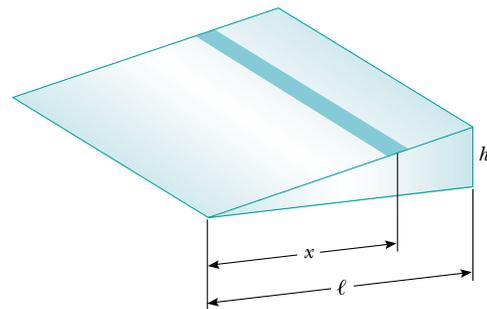


Figure P37.60

61. Consider the double-slit arrangement shown in Figure P37.61, where the slit separation is d and the slit to screen distance is L . A sheet of transparent plastic having an index of refraction n and thickness t is placed over the upper slit. As a result, the central maximum of the interference pattern moves upward a distance y' . Find y' .

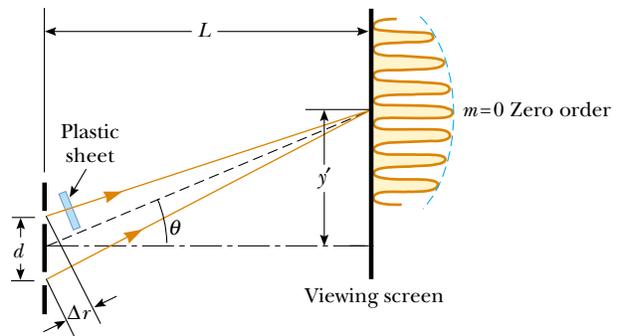


Figure P37.61

62. A plano-convex lens has index of refraction n . The curved side of the lens has radius of curvature R and rests on a flat glass surface of the same index of refraction, with a film of index n_{film} between them, as shown in Fig. 37.18a. The lens is illuminated from above by light of wavelength λ . Show that the dark Newton's rings have radii given approximately by

$$r \approx \sqrt{\frac{m\lambda R}{n_{\text{film}}}}$$

where m is an integer and r is much less than R .

63. In a Newton's-rings experiment, a plano-convex glass ($n = 1.52$) lens having diameter 10.0 cm is placed on a flat plate as shown in Figure 37.18a. When 650-nm light is incident normally, 55 bright rings are observed with the last one right on the edge of the lens. (a) What is the radius of curvature of the convex surface of the lens? (b) What is the focal length of the lens?
64. A plano-concave lens having index of refraction 1.50 is placed on a flat glass plate, as shown in Figure P37.64. Its curved surface, with radius of curvature 8.00 m, is on the bottom. The lens is illuminated from above with yellow sodium light of wavelength 589 nm, and a series of concentric bright and dark rings is observed by reflection. The interference pattern has a dark spot at the center, surrounded by 50 dark rings, of which the largest is at the outer edge of the lens. (a) What is the thickness of the air layer at the center of the interference pattern? (b) Calculate the radius of the outermost dark ring. (c) Find the focal length of the lens.

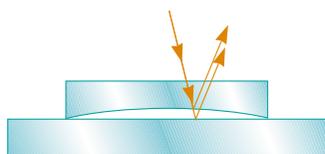


Figure P37.64

65. A plano-convex lens having a radius of curvature of $r = 4.00$ m is placed on a concave glass surface whose radius of curvature is $R = 12.0$ m, as shown in Figure P37.65. Determine the radius of the 100th bright ring, assuming 500-nm light is incident normal to the flat surface of the lens.

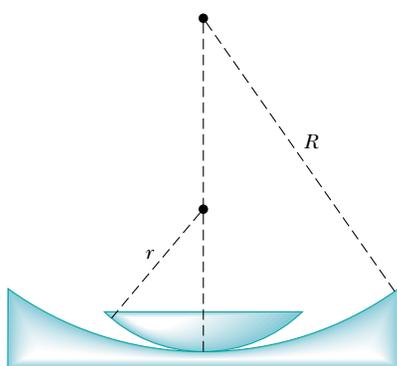


Figure P37.65

66. Use phasor addition to find the resultant amplitude and phase constant when the following three harmonic functions are combined: $E_1 = \sin(\omega t + \pi/6)$, $E_2 = 3.0 \sin(\omega t + 7\pi/2)$, and $E_3 = 6.0 \sin(\omega t + 4\pi/3)$.
67. A soap film ($n = 1.33$) is contained within a rectangular wire frame. The frame is held vertically so that the film drains downward and forms a wedge with flat faces. The thickness of the film at the top is essentially zero. The film is viewed in reflected white light with near-normal incidence, and the first violet ($\lambda = 420$ nm) interference band is observed 3.00 cm from the top edge of the film.

- (a) Locate the first red ($\lambda = 680$ nm) interference band. (b) Determine the film thickness at the positions of the violet and red bands. (c) What is the wedge angle of the film?

68. Compact disc (CD) and digital video disc (DVD) players use interference to generate a strong signal from a tiny bump. The depth of a pit is chosen to be one quarter of the wavelength of the laser light used to read the disc. Then light reflected from the pit and light reflected from the adjoining flat differ in path length traveled by one-half wavelength, to interfere destructively at the detector. As the disc rotates, the light intensity drops significantly every time light is reflected from near a pit edge. The space between the leading and trailing edges of a pit determines the time between the fluctuations. The series of time intervals is decoded into a series of zeros and ones that carries the stored information. Assume that infrared light with a wavelength of 780 nm in vacuum is used in a CD player. The disc is coated with plastic having an index of refraction of 1.50. What should be the depth of each pit? A DVD player uses light of a shorter wavelength, and the pit dimensions are correspondingly smaller. This is one factor resulting in greater storage capacity on a DVD compared to a CD.

69. Interference fringes are produced using Lloyd's mirror and a 606-nm source as shown in Figure 37.15. Fringes 1.20 mm apart are formed on a screen 2.00 m from the real source S. Find the vertical distance h of the source above the reflecting surface.

70. Monochromatic light of wavelength 620 nm passes through a very narrow slit S and then strikes a screen in which are two parallel slits, S_1 and S_2 , as in Figure P37.70. Slit S_1 is directly in line with S and at a distance of $L = 1.20$ m away from S, whereas S_2 is displaced a distance d to one side. The light is detected at point P on a second screen, equidistant from S_1 and S_2 . When either one of the slits S_1 and S_2 is open, equal light intensities are measured at point P. When both are open, the intensity is three times larger. Find the minimum possible value for the slit separation d .

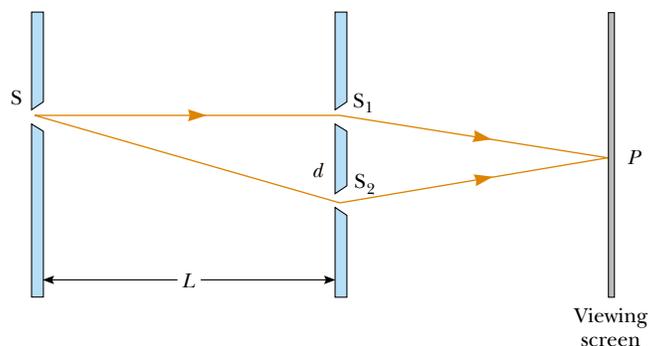
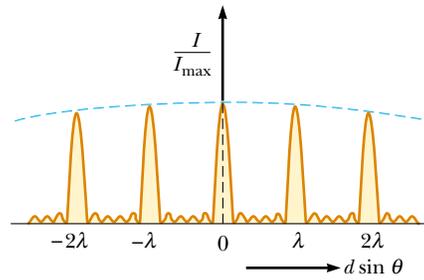


Figure P37.70

71. Slit 1 of a double slit is wider than slit 2, so that the light from 1 has an amplitude 3.00 times that of the light from 2. Show that for this situation, Equation 37.11 is replaced by the equation $I = (4I_{\max}/9)(1 + 3 \cos^2 \phi/2)$.

Answers to Quick Quizzes

37.1 (b). The geometrical construction shown in Figure 37.5 is important for developing the mathematical description of interference. It is subject to misinterpretation, however, as it might suggest that the interference can only occur at the position of the screen. A better diagram for this situation is Figure 37.2, which shows *paths* of destructive and constructive interference all the way from the slits to the screen. These paths would be made visible by the smoke.



37.2 (c). Equation 37.5, which shows positions y proportional to order number m , is only valid for small angles.

37.3 (c). Equation 37.5 shows that decreasing λ or L will bring the fringes closer together. Immersing the apparatus in water decreases the wavelength so that the fringes move closer together.

37.4 (c). Conservation of energy cannot be violated. While there is no energy arriving at the location of a dark fringe, there is more energy arriving at the location of a bright fringe than there would be without the double slit.

37.5 The graph is shown in the next column. The width of the primary maxima is slightly narrower than the $N = 5$ primary width but wider than the $N = 10$ primary width. Because $N = 6$, the secondary maxima are $\frac{1}{36}$ as intense as the primary maxima.

37.6 (a). One of the materials has a higher index of refraction than water, the other lower. For the material with a higher index of refraction, there is a 180° phase shift for the light reflected from the upper surface, but no such phase change from the lower surface, because the index of refraction for water on the other side is lower than that of the film. Thus, the two reflections are out of phase and interfere destructively.

37.7 (a). At the left edge, the air wedge has zero thickness and the only contribution to the interference is the 180° phase shift as the light reflects from the upper surface of the glass slide.