Nuclear Size

Various experiments show that the nuclear charge is uniformly distributed and the nuclear charge density ρ_c is approximately constant. Experimental evidences also show that the nuclear matter is also uniformly distributed. Hence the nuclear mass density ρ_m is also constant.

$\cancel{\rho_m \propto \frac{A}{V}}$

Now the nuclear mass is almost linearly proportional to mass number A.

$\therefore M_{nuc} \propto A$

$\rho_m \propto \frac{M_{nuc}}{V}$

or $\rho_m \propto \frac{A}{V} = \text{constant}$

$V = \frac{4}{3}\pi R^3$ [considering the nucleus a spherical one]

Hence $R^3 \propto A$

or $R \propto A^{1/3}$

$\therefore R = r_0 A^{1/3}$

r_0 = constant = nuclear radius parameter.

Nuclear charge ~~⊠~~ parameter $Z \propto A$

and nuclear charge density $\rho_c \simeq$ constant

$\rho_c \simeq \frac{Ze}{V} \simeq$ constant

Hence the ~~nuclear charge~~ distribution of nuclear charge Ze should also ~~to~~ follow the same pattern.

$\therefore$ nuclear charge radius

$\simeq$ nuclear mass radius.

The size of the nucleus was first estimated by Rutherford's experiment.

The larger the angle of scattering of alpha particle, closer its approach to the nucleus.

If the kinetic energy of the alpha particle be equal to the repulsive coulomb energy between α-particle (charge $2e$, mass m and velocity v) and the nucleus (charge Ze), it would momentarily come to rest such that the distance of closest approach R is given by

$$\frac{1}{2} m v^2 = \frac{2e \times Ze}{4\pi \varepsilon_0 R}$$

$$R = \frac{Ze^2}{\pi \varepsilon_0 m v^2}$$

~~for Z = 20,~~ $m = 6.64 \times 10^{-27}$ kg, $e = 1.6 \times 10^{-19}$ C

$v = 10^7$ m/s, $Z = 20$, $R \simeq 1.5 \times 10^{-14}$ m

The generally used methods to estimate the nuclear charge radius are

(i) Electron scattering experiment

(ii) Muonic x-ray method

(iii) Mirror nucleus method.

structure of the proton, ...

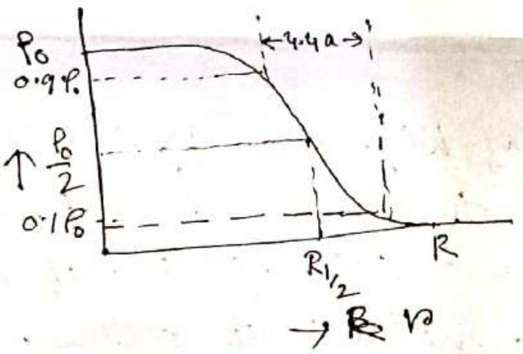
The various experimental data obtained from electron scattering experiments could be fitted well with a charge ~~distrib~~ density distribution of the form

$$\rho(r) = \frac{\rho_0}{1+\exp\{(r-R_{1/2})/a\}} \quad \text{[Fermi Distribution]}$$

ρ_0 = charge density at the centre.

$R_{1/2}$ = half density ~~value of~~ the radius

and $t = 4.4a$ is the skin thickness which is measured between 90% and 10% values of the density.



If we approximate the above distribution by a uniform charge distribution, then the corresponding equivalent radius can be written as

$$R = r_0 A^{1/3}$$

where $r_0 = 1.32\times10^{-15}$ m for $A<50$

and $r_0 = 1.21\times10^{-15}$ m for $A>50$

$a = 0.5$ fm

$t = 4.4a \approx 2.3$ fm

For the spherical nuclei with $A>15$ the charge distr. has a core of uniform density surrounded by a skin of thickness 2.3 fm. The radius of half the maximum density $R_{1/2} = 1.07A^{1/3}$.

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For $4 \leq A \leq 15$, there is no uniform core and the density decreases steadily with increasing r. There is some indication that for all nuclei there is slight diminution in the density near the centre. Further the charge density in the core region decreases somewhat as Z increases.

From different experiments the following conclusions can be arrived at:

(i) the distribution of ~~protons~~ density or protons in the nucleus is slightly different from that of all nucleons within the nucleus.,

(ii) the nuclear charge distribution is not spherically symmetric in some ~~nucleus~~; the latter ones possess electric quadrapole moment

(iii) the electrical radius of a nucleus is not significantly less than the radius of nuclear matter consisting of all the nucleons.

$\rho_N \simeq 2 \times 10^{17}\ kg/m^3$

atomic density
$\rho_A \simeq 2 \times 10^{3}\ kg/m^3$

~~Grea~~ Static charge characteristics → ground state nucleus
Dynamic characteristics are exhibited in the process of nuclear reactions, nuclear excitations such nuclear decay.

Nucleus → Z protons + N neutrons
↓
atomic number

$Z + N = A$ → Mass number

Same Z, Different A → isotope
Same A, different Z → isobar
Same N, different A → isotone

isotope → stable isotope, unstable isotope

hydrogen → $\underbrace{\text{Hydrogen } (^1H), \text{ deuterium } (^2H)}_{\text{stable}}$
tritium (^3H) unstable

Elements having more than one stable ~~isope~~ isotope ~~, occur~~ in natural state are mixtures of these isotopes in fixed properties, known as their relative abundances, which remain more or less the same, irrespective of their sources.

$Li \rightarrow {}^6Li + {}^7Li$ $(P=3)$
7.4% 92.6%

$H_2 \rightarrow {}^1H + {}^2H$
$(99.99\%) + (0.01\%)$

By mass spectroscope one can not measure the nuclear mass but the atomic mass

$$M_{nuc} = M(A, Z) - Z m_e$$ (ignoring the binding energy of electrons)

Nuclons are ~~at~~ strongly bound requires energy of a few MeV to ~~to~~ break away a nuclion from a nucleus.

Binding engy - definition

Energy evolved to form a nuclus by combining Z protons and N neutrons ~~a fraction of the total mas~~

a fraction of the total mass disappears

$$E_B = \Delta M c^2$$

$$\Delta M = Z M_H + N M_n - M(A,Z)$$

$$E_B = \left[Z M_H + N M_n - M(A,Z)\right] c^2$$

$$= \left[Z M_H + N M_n - M(A,Z)\right]$$ (in energy units)

unit of atomic mass

unified ~~unit of~~ atomic mass unit

$= 1 u.$

$= \frac{1}{12} \times$ ~~a~~ $\underbrace{\text{mass of the atom of } {}^{12}C}_{12}$

$=$

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1 mole of $^{12}C = 12\,g = 12\times10^{-3}\,kg$

1 mole = N_0 number of atoms

$= 6.02205\times10^{23}$ " " "

the mass of each ^{12}C atom is

$$= \frac{12\times10^{-3}}{6.02205\times10^{23}}\,kg$$

$$= 12\times1.660566\times10^{-27}\,kg$$

$$1u = \frac{1}{12}\times12\times1.660566\times10^{-27}\,kg$$

$$= 1.660566\times10^{-27}\,kg.$$

$$= 1.660566\times10^{-27}\times c^2$$

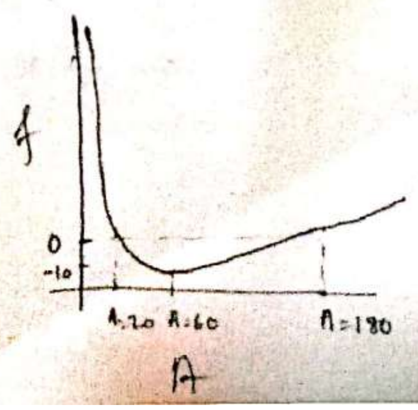
$$= 931.502\,MeV.$$

Atom	Atomic mass (u)	Mass defect (u)
^1n	1.008665	+ 0.008665
^1H	1.007825	0.007825
^{12}C	12	0
^{16}O	15.994915	− 0.005085
^{197}Au	196.96654	−0.03346
^{238}U	238.05082	+0.05082

very small deviation of Atomic masses from the corresponding mass numbers.

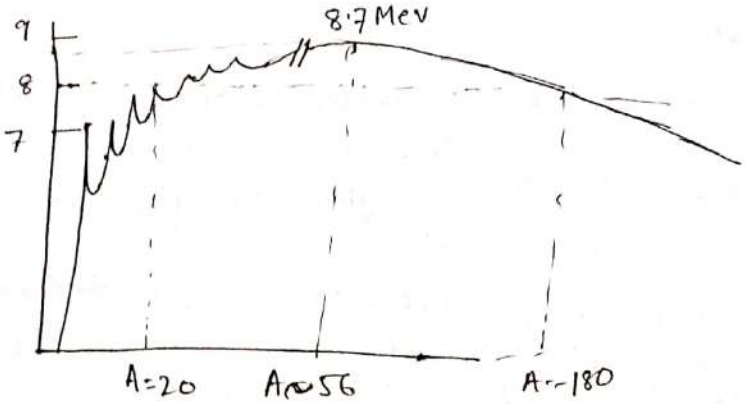
$\Delta M = M(A,Z) - A$
↓
Mass defect

$$\text{Packing fraction} = \frac{\Delta M}{A}$$



Binding fraction

$$f_B = \frac{E_B}{A} = \frac{ZM_H + NM_n - M(A,Z)}{A}$$



$20 < A < 180$ $f_B \sim 8.5$ MeV

for heaviest nuclei $f_B = 7.5$ MeV

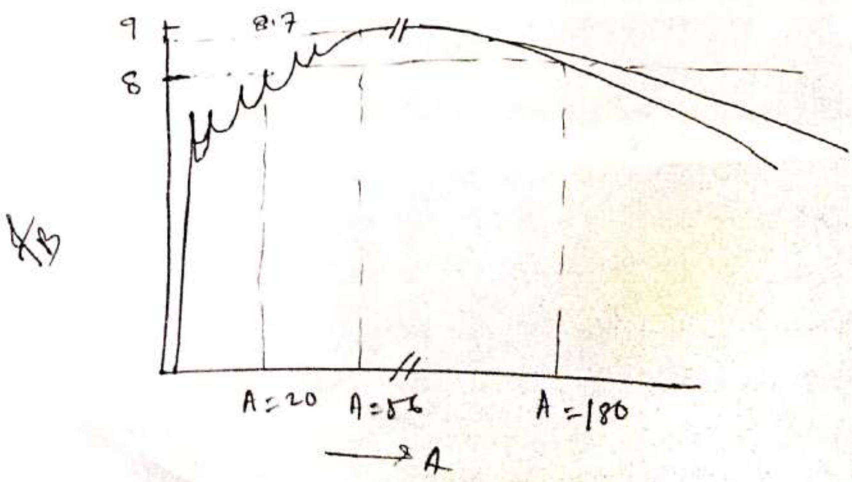
peaks → ^4He ^8Be ^{12}C ^{16}O $A = 4n$

less prominent peaks → Z or N $= \underbrace{20, 28, 50, 82, 126}_{\text{magic numbers}}$

$$f_B = \frac{Zf_H + Nf_n}{A} - f \qquad f_H = M_H - 1$$

$$f_n = M_n - 1$$

∴ f_B and f are ~~comple~~ complementary to each other



a) f_B for the very light nuclei is very small and rises rapidly with A attaining a value of ~ 8 MeV/nucleon for $A \sim 20$. It then rises slowly with A and attains a maximum of 8.7 MeV per nucleon at $A \sim 56$. For higher A, it decreases slowly.

b) For $20 < A < 180$, the variation of f_B is very slight, so that it may be taken to be approximately constant in this region having a mean value of ~ 8.5 MeV per nucleon.

c) For very heavy nuclei $(A > 180)$, f_B decreases monotonically with the increase of A. For the heaviest nuclei, f_B is about 7.5 MeV/nucleon.

d) For very light nuclei, there are rapid fluctuations in the values of f_B. In particular, peaks are observed in the (f_B vs. A) graph for the even-even nuclei ^4He, ^8Be, ^{12}C, ^{16}O etc., for which $A = 4n$ where n is an integer. Similar, but less prominent peaks are observed at the values of Z or N equal to 20, 28, 50, 82 and 126. These are known as magic numbers.

Nuclear Spin

① Like electrons, protons and neutrons also possess intrinsic spin angular momenta.

② Since the nucleus is a bound ~~system~~ quantum system, the nucleons also possess quantized orbital angular momenta about the centre of mass of the nucleus.

The resultant angular momentum $\vec{I}$ of the nucleus is given by

$$\vec{I} = \vec{L} + \vec{S}$$

$$|\vec{I}| = \sqrt{I(I+1)}\,\hbar$$

$$|\vec{L}| = \sqrt{L(L+1)}\,\hbar$$

$$|\vec{S}| = \sqrt{S(S+1)}\,\hbar$$

During measurement, it is the largest component of the angular momentum ~~do~~ along the direction of the applied electric or magnetic field which is determined. For the three cases mentioned above, these have the magnitude I, L, and S respectively in units of $\hbar$.

The resultant spin angular momentum of the nucleus is given by the vector sum of the spins of the individual nucleons

$$\vec{S} = \sum_i \vec{s}_i$$

Similarly $\vec{L} = \sum \vec{l}_i$

$s_i = \frac{1}{2}$ (in units of $\hbar$)

S can be either integral or half-integral depending on whether the number of nucleons A in the nucleus is even or odd.

Since l_i is integral (0, 1, 2, etc.), L is integral or zero.

$\therefore$ I = integral [for A even]

= half-integral [for A odd]

$I \rightarrow$ nuclear spin

For even Z or even N $I = 0$ at ground state

$\Rightarrow$ There is a tendency of the nucleons inside the nucleus to form pairs with equal and oppositely aligned angular momenta, which cancel out in pairs for like nucleons.

Majority of the nucleons of either type seems to form a core in which even members of protons and neutrons are grouped in pairs of zero spin and orbital angular momenta so that the core itself has zero total angular momentum. The few remaining nucleons outside the core determine the nuclear spin which is thus a small number, integral or half-integral.

Nuclear magnetic moment

Like the electron, the proton and the neutrons possess intrinsic magnetic dipole moments.

$$\mu_p = 2.7927\ \mu_N$$

$$\mu_n = -1.9131\ \mu_N$$

$$\mu_N = \frac{e\hbar}{2M_p} = \text{nuclear magneton.}$$

$$\mu_N = \frac{\mu_B}{1836} = 5.0571 \times 10^{-27} J/T.$$

∴ proton and neutron magnetic moments are of the order of 10^{-3} times the electronic magnetic moment, which is equal to the Bohr magneton.

Since the nuclei are made up of protons and neutrons, the magnetic moments of ~~of~~ the nuclei are also much smaller than the atomic magnetic moments.

For the protons, because of its positive charge, the direction of $\vec{\mu}_p$ is the same as that ~~of~~ of ~~$\vec{S}_p$~~ $\vec{S}_p$ where $\vec{S}_p$ is the proton spin ~~vector~~. angular momentum vector

The negative sign of μ_n indicates that $\vec{\mu}_n$ is directed opposite to $\vec{S}_n$.

(Though, the neutron is a neutral particle, still it possesses a negative magnetic moment. It can be attributed to the ~~neo~~ mesonic exchange between neutron and proton inside the nucleus.

Alpha disintegration Energy

Residual nuclei (M_1, v_1)

$${}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He$$

$$M_\alpha v_\alpha = M_1 v_1$$

α-disintegration energy (total energy released in disintegration process),

$$Q = \frac{1}{2} M_\alpha v_\alpha^2 + \frac{1}{2} M_1 v_1^2$$

$$= \frac{1}{2} M_\alpha v_\alpha^2 + \frac{1}{2} \frac{M_\alpha^2 v_\alpha^2}{M_1}$$

$$= \frac{1}{2} M_\alpha v_\alpha^2 \left[1 + \frac{M_\alpha}{M_1}\right]$$

$$M_1 v_1^2 = \frac{M_1^2 v_1^2}{M_1} =$$

K.E of α-particle $E_\alpha = \frac{1}{2} M_\alpha v_\alpha^2$

$$Q = E_\alpha \left[1 + \frac{M_\alpha}{M_1}\right] = E_\alpha \frac{M_1 + M_\alpha}{M_1}$$

masses of the nuclei in the unit of atomic masses are close to their mass numbers.

$$M_1 \simeq A - 4 \quad M_\alpha \simeq 4$$

$$Q = E_\alpha \frac{A}{A-4}$$

$E_\alpha \rightarrow$ measured experimentally

$Q \rightarrow$ can easily be calculated.

$^{210}Po \rightarrow E_\alpha = 5.305$ MeV

$Q = 5.408$ MeV.

Importance of accurate measurement of α-disintegration energy

A part of the mass of the disintegrating nucleus is ~~so~~ converted into energy as the α-disintegration energy.

$$M > M_\alpha + M_1$$

$$Q = (M - M_\alpha - M_1)c^2$$

all the masses are atomic masses.

The mass of the ~~sat~~ stable isotopes can be measured accurately by the mass spectroscopes. However, ~~it fails~~ this is not applicable to radioactive isotopes. In the case of α-emitters, atomic mass of the ~~α~~ disintegrating ~~nucleus~~ atom can be measured accurately by accurate measurement of α-disintegration energy.

Range of α-particles

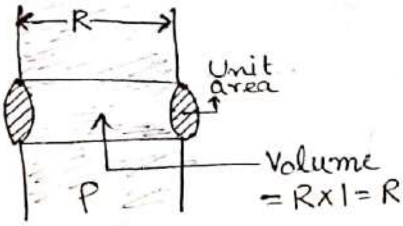
Range: The distance through which an α-particle travels in a specified material before stopping to ionise it is called its range in that material. The range is thus a sharply defined ionisation path-length.

The range is ~~highter~~ highest in gases, less in liquids and the least in solids due to ~~more~~ more and more dense packing of the particles.

In a gas, the range depends on (i) the initial energy of the α-particle, (ii) the ~~ion~~ ionisation potential of the gas and (iii) the chances of collision between the α-particles and the gas particles, i.e., on the nature and the temperature and pressure of the gas.

Measured in unit of length, the range in solids and liquids is very small ~ 10^{-3}mm for α of few MeV. In a gas, it is few cm in length.

The range can also be expressed in the ~~un~~ unit of the mass of the substance traversed.



We draw a cylinder of unit cross-sectional area and length equal to the range R within the substance, the amount of matter contained within it is $R\rho$, where ρ is the density of the material. Thus the unit of the range in this case is mass per unit area (kg/m^2). Expressed in this unit, the range is of the same order of magnitude in different materials, solid, liquid or gas.

<u>Range-energy relationship for α-particles</u>

From experimental studies, it was observed

$$R \propto v^3$$

$$\text{or } R = a v^3 \longrightarrow \text{Geiger law}$$ | valid only in limited velocity range.

a = constant.

$$E = \frac{1}{2} m v^2$$

$$R \propto E^{3/2} = b\, E^{3/2}$$ } empirical law of Geiger

$$a = 9.416 \times 10^{-24}$$

$$b = 3.15 \times 10^{-3}$$

<u>Specific ionisation</u>

Due to ionisation, there will be a large number of ion-pairs generated along the path of the α-particles in a gas. Their number in unit length of the path at any point is proportional to the energy lost in the region.

The number of ion-pairs formed per unit path-length at any point in the path of the α-particle is called specific ionisation.

$$E \propto R^{2/3}$$

$$\frac{dE}{dR} \propto R^{1/3} \propto \frac{1}{v} \quad (\because R \propto v^3)$$

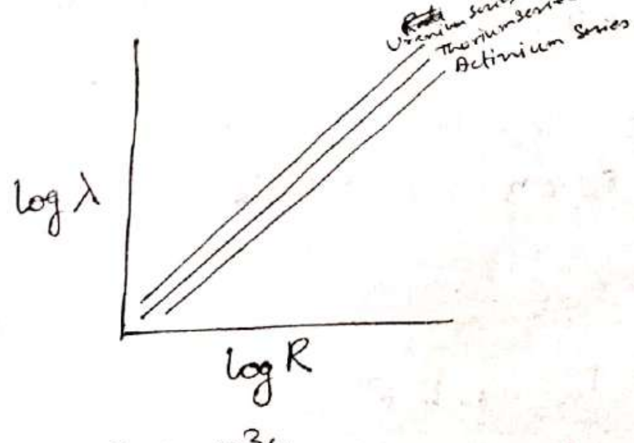
Thus the ionisation produced by an α-particle at any point ~~ie~~ in its track is ~~so~~ inversely proportional to its velocity at that point.

Geiger-Nuttall law

~~The~~ Geiger and Nuttall discovered an empirical relationship between the ranges of the α-particles and the disintegration constants of the naturally α-active substances emitting them. This is known as Geiger-Nuttall law which can be expressed as

$$\log \lambda = A + B \log R$$

where A and B are constants.



B is same for all the series

$\because R \propto E^{3/2}$

$$\therefore \log \lambda = C + D \log E$$

C, D $\rightarrow$ constants

$$T_{1/2} = \frac{\ln 2}{\lambda}$$

$$\text{or } \lambda = \frac{\ln 2}{T_{1/2}} = \frac{0.693}{T_{1/2}}$$

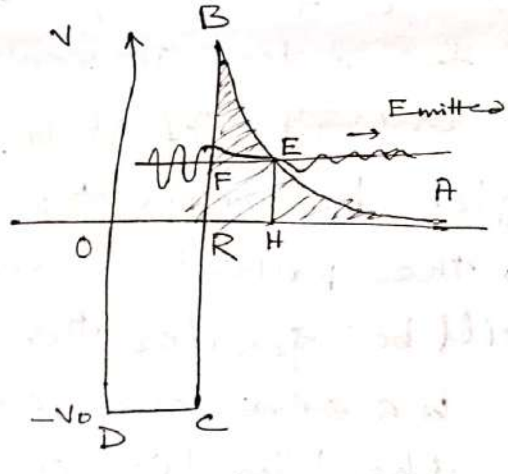
$$\log\left[\frac{0.693}{T_{1/2}}\right] = A + B \log R$$

$$\log(0.693) - \log T_{1/2} = A + B \log R$$

$$\log T_{1/2} = C - B \log R$$

α- disintegration

neutrons and protons forming the nucleus of an atom are strongly attracted to one another. The attractive force acts upto distances of the order of 2×10^{-15} m. Two ~~pr~~ protons and ~~a~~ two neutrons sometimes form a cluster ~~known~~ known as the α- particle. As long as the α-particle is within the nucleus, ~~it~~ it must be acted upon by this strong short range ~~specifically~~ nuclear attractive force. However, once it is outside the nucleus, the Coulomb repulsion due to the residual nucleus of positive charge $(Z-2)$ acts on it.



outside the nucleus, the Coulomb potential energy $V_c = \dfrac{2(Z-2)e^2}{4\pi\varepsilon_0 r}$ is +ve.

For $r < R$, the nuclear radius, this suddenly changes into an attractive potential. Since, it is a strong short range ~~at~~ attractive potential, it must have a very steep positive slope.

The transition from the repulsive Coulomb potential to the attractive potential in the nucleus takes place at the nuclear surface $r = R$ where the Coulomb potential has the ~~last~~ largest value

$$V_S = \frac{2(Z-2)e^2}{4\pi \varepsilon_0 R}$$

$${}^{226}_{88}Ra \longrightarrow {}^{222}R_n + {}^{4}_{2}He$$

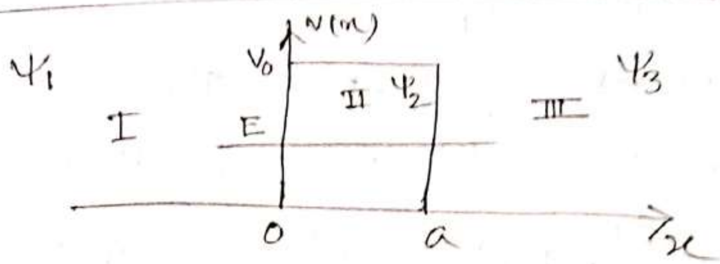
$$V_S = 34 \text{ MeV}$$

$$Q_\alpha = 4.88 \text{ MeV}. \quad Q_\alpha << V_S$$

According to ~~cla~~ classical mechanics, the value of Q_α should, at least, be equal to V_S ~~to~~ either to escape from the nucleus or to enter it ~~for~~ from ~~the~~ outside. ~~its energy~~ If it is lower, then in some region between the two curves AB and BC, the potential energy of the α-particle will be greater than its total energy. This is a ~~cla~~ classically forbidden region, since the kinetic energy of the α-particle would be negative in this region. ~~Ex~~ Q_α is represented by the line FE. As long as the α-particle is inside the nucleus or at points to the right of EH outside the nucleus, the potential energy is less than its total energy, so that its kinetic energy is positive. However in the region RH, known as the potential barrier region, the total energy of the α-particle is less than its potential energy. The kinetic energy is negative here, so that this is the also classically forbidden region. An α-particle in this region can neither escape nor enter into the nucleus.

However, quantum mechanically an α-particle initially inside the nucleus has a finite probability of coming out of it.

One dimensional potential barrier



$$V(x) = 0 \qquad x < 0$$
$$= V_0 \qquad 0 \le x \le a$$
$$= 0 \qquad x > a$$

$$\frac{d^2\psi_1}{dx^2} + \frac{2mE}{\hbar^2} E\psi_1 = 0 \quad \text{—— (I)}$$

$$\frac{d^2\psi_2}{dx^2} + \frac{2m}{\hbar^2}(E - V_0)\psi_2 = 0 \quad \text{—— (II)}$$

$$\frac{d^2\psi_3}{dx^2} + \frac{2mE}{\hbar^2}\psi_3 = 0 \quad \text{—— (III)}$$

$$\text{(I)} \Rightarrow \quad \psi_1'' + \alpha^2\psi_1 = 0 \quad \text{—— (I)}$$

$$\psi_2'' - \beta^2\psi_2 = 0 \quad \text{—— (II)}$$

$$\psi_3'' + \alpha^2\psi_3 = 0 \quad \text{—— (III)}$$

$$\alpha^2 = \frac{2mE}{\hbar^2} \qquad \beta^2 = \frac{2m(V_0 - E)}{\hbar^2}$$

$$\psi_1 = Ae^{i\alpha x} + Be^{-i\alpha x}$$

$$\psi_2 = De^{-\beta x} + Fe^{\beta x}$$

$$\psi_3 = Ce^{i\alpha x}$$

B.C.

(i) $\psi_1\big|_{x=0} = \psi_2\big|_{x=0}$

(ii) $\psi_1'\big|_{x=0} = \psi_2'\big|_{x=0}$

(iii) $\psi_2\big|_{x=a} = \psi_3\big|_{x=a}$

(iv) $\psi_2'\big|_{x=a} = \psi_3'\big|_{x=a}$

$$A+B = D+F \quad \text{— (i)}$$

$$i\alpha(A-B) = -\beta(D-F)$$

$$\text{or } A-B = \frac{i\beta}{\alpha}(D-F) \quad \text{— (ii)}$$

$$De^{-\beta a} + Fe^{\beta a} = Ce^{i\alpha a} \quad \text{— (iii)}$$

$$-\beta\left[De^{-\beta a} - Fe^{\beta a}\right] = i\alpha C e^{i\alpha a} \quad \text{— (iv)}$$

$$D = \frac{C}{2\beta}(\beta - i\alpha).\, e^{i\alpha a} e^{\beta a}$$

$$\cancel{F = \frac{C}{2\beta} e^{i\alpha a}\left(1+\frac{i\alpha}{\beta}\right)}$$

$$F = \frac{C}{2\beta}(\beta + i\alpha) e^{i\alpha a} e^{-\beta a}$$

$$A = \frac{C}{4} e^{i\alpha a}\left[-\frac{(\beta - i\alpha)^2}{i\alpha\beta} e^{\beta a} + \frac{(\beta+i\alpha)^2}{i\alpha\beta} e^{-\beta a}\right]$$

$$B = \frac{C}{4} e^{i\alpha a}\left[\frac{\beta^2+\alpha^2}{i\alpha\beta} e^{\beta a} - \frac{\beta^2+\alpha^2}{i\alpha\beta} e^{-\beta a}\right]$$

The incident probability current density

$$j_{in} = \frac{i\hbar}{2m}\left[\psi_{in}\frac{d\psi_{in}^*}{dx} - \psi_{in}^*\frac{d\psi_{in}}{dx}\right]$$

$$\psi_{in} = Ae^{i\alpha x} \qquad \frac{d\psi_{in}}{dx} = i\alpha\psi_{in}$$

$$\psi_{in}^* = A^* e^{-i\alpha x} \qquad \frac{d\psi_{in}^*}{dx} = -i\alpha\psi_{in}^*$$

$$j_{in} = \frac{i\hbar}{2m}\left(-i\alpha|\psi_{in}^{*}|^2 - i\alpha|\psi_{in}|^2\right]$$

$$= \frac{\hbar\alpha}{m}|A|^2$$

The transmitted probability current density is

$$j_t = \frac{i\hbar}{2m}\left(\psi_t \frac{d\psi_t^*}{dx} - \psi_t^* \frac{d\psi_t}{dx}\right)$$

$$\psi_t = \psi_3 = C e^{i\alpha x}$$

$$\psi_t^* = C^* e^{-i\alpha x}$$

$$\frac{d\psi_t}{dx} = i\alpha\psi_3 \qquad \frac{d\psi_t^*}{dx} = -i\alpha\psi_3^*$$

$$J_t = \frac{i\hbar}{2m}\left(-i\alpha|\psi_3|^2 - i\alpha|\psi_3|^2\right)$$

$$= \frac{\hbar\alpha}{m}|C|^2$$

Transmission coeff.

$$T = \frac{j_t}{j_{in}} = \frac{|C|^2}{|A|^2}$$

$$= \frac{4\alpha^2\beta^2}{(\alpha^2-\beta^2)\sinh^2\beta a + 4\alpha^2\beta^2\cosh^2\beta\alpha}$$

$$= \frac{4\alpha^2\beta^2}{4\alpha^2\beta^2 + (\alpha^2+\beta^2)^2\sinh^2\beta a}$$

$$= \left\{1 + \frac{(\alpha^2+\beta^2)^2}{4\alpha^2\beta^2}\sinh^2\beta a\right\}^{-1}$$

$$(\alpha^2+\beta^2)^2 = \left(\frac{2mV_0}{\hbar^2}\right)^2 \qquad \alpha^2\beta^2 = \left(\frac{2m}{\hbar^2}\right)^2 E(V_0 - E)$$

$$T = \left\{1 + \frac{V_0^2}{4E(V_0-E)} \sinh^2 \beta a\right\}^{-1}$$

Classically $T=0$

but quantum mechanically $T \neq 0$.

As E decreases below V_0, T monotonically decreases because of the factor $E(V_0-E)$. Also as the thickness a of the barrier increases, $\sinh \beta a$ increases rapidly and hence T decreases.

For $\beta a >> 1$ $\sinh^2 \beta a \sim \frac{1}{4} e^{2\beta a} >> 1$

$$T = \frac{16 E(V_0-E)}{V_0^2} e^{-2\beta a}$$

Ex

$V_0 = 15$ MeV

$a = 2 \times 10^{-14}$ m.

$E = 5$ MeV

$M = M_\alpha = 4 \times 1.66 \times 10^{-27}$ kg.

$$\beta^2 = 2 \cdot \frac{M}{\hbar^2}(V_0-E)$$

$= 1.927 \times 10^{30}$

$\beta = 1.388 \times 10^{15}$

$2\beta a = 55.53$

$T = 2.55 \times 10^{-24}$

The particle has the probability of about 2.5 in 10^{24} collisions against the barrier wall to penetrate through the barrier.

Energetics of β-decay

β⁻ decay

$$ {}^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + \beta^- $$

β⁺ decay

$$ {}^{A}_{Z}X \rightarrow {}^{A}_{Z-1}Y + \beta^+ $$

Orbital electron capture

$$ {}^{A}_{Z}X + e^- \rightarrow {}^{A}_{Z-1}Y $$

$$ Q_{\beta^-} = [M_n(A,Z) - M_n(A,Z+1) - m_e]c^2 $$
$$ = [M(A,Z) - Zm_e - M(A,Z+1) + (Z+1)m_e - m_e]c^2 $$
$$ = [M(A,Z) - M(A,Z+1)]c^2 $$
$$ = [M(A,Z) - M(A,Z+1)] \quad [c=1]. $$

$$ Q_{\beta^-} > 0 \quad \text{if } M(A,Z) > M(A,Z+1) $$

$$ Q_{\beta^+} = [M_n(A,Z) - M_n(A,Z-1) - m_e]c^2 $$
$$ = [M(A,Z) - Zm_e - M(A,Z-1) + (Z-1)m_e - m_e]c^2 $$
$$ = [M(A,Z) - M(A,Z-1)]c^2 - 2m_e c^2 $$

$$ Q_{\beta^+} > 0, \quad \text{if } M(A,Z) > M(A,Z-1) + 2m_e $$

rest energy of electron → 0.511 MeV

The mass of the parent atom must be greater than that of the daughter atom by an amount at least equal to 1.022 MeV.

$$Q_e = [M_n(A,Z) + m_e - M_n(A, Z-1)]c^2 - B_e$$

$$= [M(A,Z) - Zm_e + m_e - M(A, Z-1) + (Z-1)m_e]c^2 - B_e$$

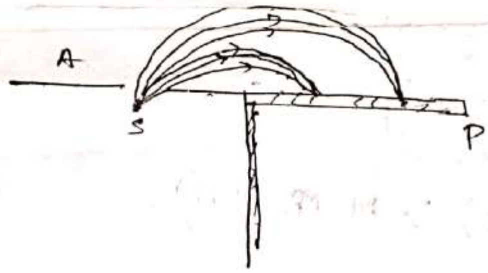
$$= [M(A,Z) - M(A, Z-1)]c^2 - B_e$$

$$= M(A,Z) - [M(A,Z-1) + B_e]$$

$Q_e > 0$ if $M(A,Z) > M(A,Z-1) + B_e$

Electron capture decay and β^+ decay can occur in one and the same nucleus if to Q_e and $Q_{\beta^+} > 0$ are satisfied.

Determination of the β-energy



magnetic spectrometer

β-particles with continuous distribution of velocities. → continuous spectrum

energy of β-particles — 0 → a maximum.

intensity of blackening is different at different points → intensities of the β-particles of different velocities are different.

$$\frac{mv^2}{r} = Bev$$

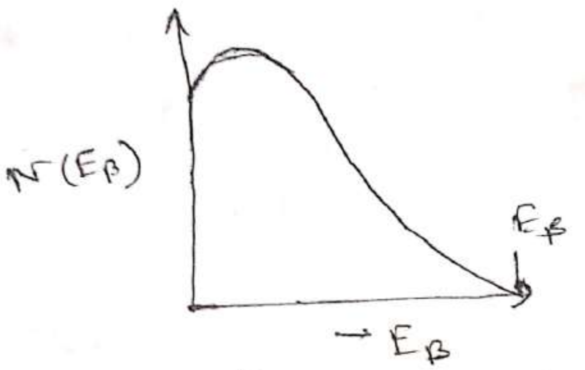
$$p = mv = Ber$$

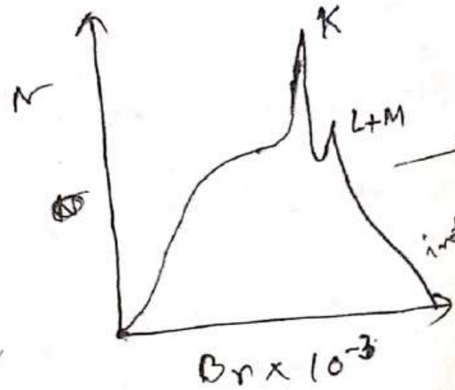
Total energy

$$W^2 = p^2c^2 + m_0^2c^4$$

K.E. of β-particle $E_\beta = W - m_0c^2$

$$= \sqrt{B^2e^2r^2c^2 + m_0^2c^4} - m_0c^2$$





→ sharp peaks due to ~~internal conversion electrons~~ electron groups of certain definite energies

→ internal conversion electrons

$N(E_\beta)dE_\beta$ is the number of β-particles in the energy range E_β to $E_\beta + dE_\beta$

$N(E_\beta) \rightarrow$ number of β-particles in unit energy interval at E_β.

Difficulties in interpretation of β-spectrum

(1) The nuclear energy states are discrete. So the continuous β-spectrum appears to be a violation of the conservation of energy principle.

(6) The β-emission from a radioactive nuclide is supposed to be the result of the transformation of either (a) or (b) below

$$a)\ p \rightarrow n + \beta^{+}$$

$$b)\ n \rightarrow p + \beta^{-}$$

Since all the particles in the above two transformations are known to have half-integral spin, the ~~to~~ law of conservation of angular momentum is also violated.

(2) When a β-particle is emitted with an energy E_β, the difference $E_m - E_\beta$ cannot be accounted for.
maximum β-energy

(3) By placing the decaying nuclei in a delicate calorimeter with thick wall that ~~st~~ stops the β-particles, the heat produced is related to the average energy and ~~to~~ not to the maximum energy. The missing energy is thus not absorbed in matter immediately surrounding the emitter.

(4) Attempts to explain the observed continuous distribution by ~~a~~ way of loss of different amounts of energy due to scattering of β-particles in the source also failed.

(5) The emitted β-particles also does not travel in the direction opposite to the recoil velocity of the daughter product. It thus appears to violate the principle of conservation of linear momentum also.

Neutrino hypothesis of Pauli

To explain these apparent inconsistencies, Wolfgang Pauli, in 1930 proposed that at the time of β-decay of a nucleus, a hitherto unobserved second particle, in addition to the electron, is emitted, which carries away the balance of energy $E_\nu = E_m - E_\beta$ so that the total energy of the two particles is equal to the maximum β-energy E_m. When the electron is emitted with zero kinetic energy, the second particle is emitted with the maximum energy $E_\nu = E_m$. On the other hand, when the electron is emitted with the energy E_m, the other particle has energy $E_\nu = 0$.

To conserve the charge in the process, the new particle should be electrically neutral. Further, the β-particle itself is capable, on occassions of carrying off all the available energy. So the new particle should carry little kinetic energy and, in fact, would have to have exceedingly small rest mass. Pauli postulated that the new particle had zero rest mass as well as zero charge.

To conserve the momentum, the new particle should have a spin equal to $\frac{1}{2}\hbar$. Further the new particle must interact very weakly with matter.

The new particle was labelled neutrino and symbolised by ν.

The neutrino hypothesis explains well the emission of β-particles by transition of nucleon from neutron to the proton state with the simultaneous creation of an electron-neutrino pair. These two particles escape with a constant total energy, the maximum energy E_m ~~avali~~ available, being equal to the difference between the energies of the original and the final nucleus.

Subsequently, it was seen that there are, in fact, two kinds of neutrino involved in β-decay — the neutrino (ν) and the antineutrino ($\bar{\nu}$).

$$n \rightarrow p + e^- + \bar{\nu}$$

$$\text{Ex.} \quad {}^3_1H \rightarrow {}^3_2He + e^- + \bar{\nu}$$

$$p \rightarrow n + e^+ + \nu$$

$$\text{Ex.} \quad {}^{11}_6C \rightarrow {}^{11}_5B + e^+ + \nu$$

Properties of a neutrino

(1) chargeless
(2) The mass of the neutrino should be zero or very nearly so.
(3) Intrinsic spin $\frac{1}{2}$.
(4) obeys F-D statistics
(5) possesses very small magnetic moment
(6) It interacts very weakly with matter.

Parity

~~$V(\vec{r})$~~

particle mass = m, the wave function associated with it is $\psi(\vec{r})$

moves in a potential $V(\vec{r})$

Reflection or parity operator $\hat{P}$

Potential is symmetric about $|\vec{r}|=0$

$$V(-\vec{r}) = V(\vec{r})$$

$$\hat{P}\, V(\vec{r}) = V(-\vec{r}) = V(\vec{r})$$

Time independent Schrodinger equation is

$$\hat{H}\psi(\vec{r}) = E\psi(\vec{r}) \quad \text{—①}$$

$\hat{H}(\vec{r})$ Hamiltonian $= -\frac{\hbar^2}{2m}\nabla^2 + V(\vec{r})$

On reflection $\vec{r} \longrightarrow -\vec{r}$

$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ remains unchanged

Since, $V(\vec{r})$ also remains unchanged

So, $\hat{H}(\vec{r})$ remains invariant under reflection

$\hat{P}\hat{H}(\vec{r}) = \hat{H}(-\vec{r}) = \hat{H}(\vec{r})$

~~$\hat{P}\hat{H}(\vec{r}) = \hat{H}(-\vec{r}) = \hat{H}(\vec{r})$~~

$\hat{P}\hat{H}(\vec{r})\psi(\vec{r}) = \hat{P}E\psi(r)$

$$\hat{H}(-\vec{r})\psi(-\vec{r}) = E\psi(-\vec{r})$$

$\hat{P}\hat{H}(\vec{r})\psi(\vec{r}) = \hat{H}(\vec{r})\psi(-\vec{r})$

$\hat{P}E\psi(\vec{r}) = E\hat{P}\psi(\vec{r}) = E\psi(-\vec{r})$

or $\hat{H}(\vec{r})\psi(-\vec{r}) = E\psi(-\vec{r}) \quad \text{—②}$

Hence, $\psi(\vec{r})$ and $\psi(-\vec{r})$ are eigenfunctions belonging to same eigenvalue E

If $\psi(\vec{r})$ and $\psi(-\vec{r})$ are non-degenerate, i.e. they are linearly dependent one can write

$$\psi(-\vec{r}) = \lambda\psi(\vec{r})$$

$$\hat{P}\psi(\vec{r}) = \psi(-\vec{r}) = \lambda\psi(\vec{r})$$

$$\hat{P}^2\psi(\vec{r}) = \hat{P}\hat{P}\psi(\vec{r}) = \hat{P}\psi(-\vec{r}) = \psi(\vec{r})$$

$$\hat{P}\hat{P}\psi(\vec{r}) = \hat{P}\lambda\psi(\vec{r}) = \lambda\hat{P}\psi(\vec{r}) = \lambda\psi(-\vec{r}) = \lambda^2\psi(r)$$

$\psi(r) = \lambda^2\psi(r)$

$\psi(-\vec{r}) = \pm\psi(\vec{r})$

$\lambda^2 = 1 \qquad \lambda = \pm 1$

$\psi(-r) = \psi(r) \rightarrow$ symmetric, even parity

$\psi(-r) = -\psi(r) \rightarrow$ antisymmetric, odd parity

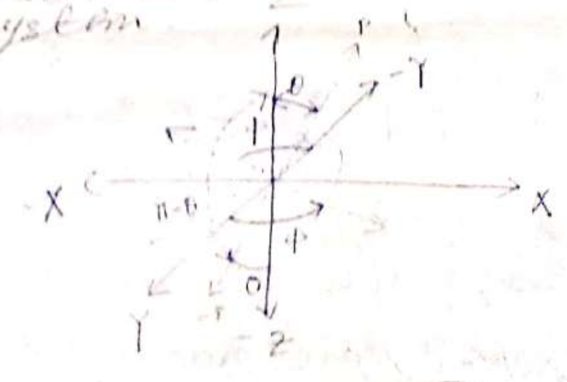
$\hat{P}\psi(\vec{r}) = +1\psi(\vec{r}) \rightarrow$ even parity

$\hat{P}\psi(\vec{r}) = -1\psi(\vec{r}) \rightarrow$ ~~even~~ odd parity

If $\psi(\vec{r})$ and $\psi(-\vec{r})$ are degenerate one can take linear combination of $\psi(\vec{r})$ and $\psi(-\vec{r})$

e.g. $\underbrace{[\psi(\vec{r}) + \psi(-\vec{r})]}_{\text{even parity}}$ and $\underbrace{[\psi(\vec{r}) - \psi(-\vec{r})]}_{\text{odd parity}}$

This ~~co~~ classification of parity is possible only if the Hamiltonian $\hat{H}$ remains invariant under parity operation, i.e. the interaction potential $V(\vec{r})$ remains unchanged by parity operation. The parity thus defined depends on quantum ~~&~~ state of motion of the system



For a particle under a central force (i.e. ~~by~~ hydrogen-like atoms) the parity ~~is determined by the~~ ~~so~~ square of the orbital angular momentum operator $\hat{L}^2$ can be ~~co~~ classified according to parity. The eigenfunctions are ~~rf~~ represented by spherical harmonics $Y_l^m(\theta, \phi)$.

In spherical polar co-ordinates, ~~the~~

$$\hat{P}\, Y_l^m(\theta,\phi) = Y_l^m(\pi-\theta, \pi+\phi)$$

$$= (-1)^l\, Y_l^m(\theta,\phi)$$

Hence, if the azimuthal quantum number $l = 0$ or even, the eigenfunctions of $\hat{L}^2$ remains unchanged by parity operation; i.e. they have even parity. If l is odd then the parity of the eigenfunctions is odd.

parity $\pi = (-1)^l$

For system of particles,

The parity $\pi = (-1)^L$

where $L = \sum l$, the orbital angular momentum of the system.

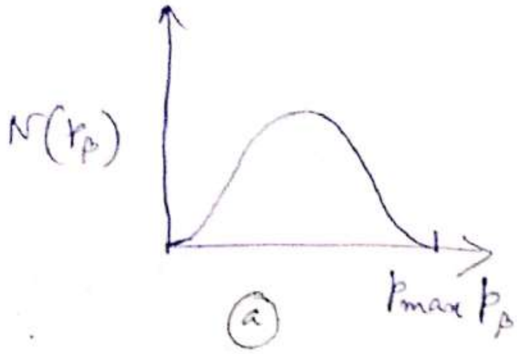
Fundamental particles may possess intrinsic parities. → related to the inversion of some internal axis. of the particle.

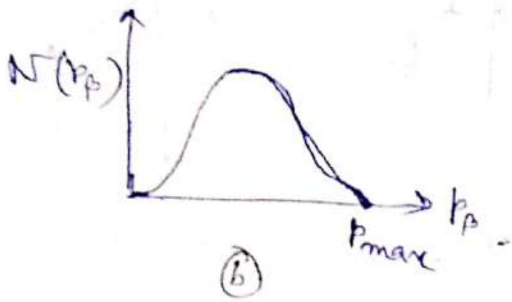
Total parity → orbital parity x intrinsic parity.

parity is actually defined in a relative manner

nucleons → even parity (convention).

Neutrino mass





The theoretical shape of the β-distribution curve near the end-point $E_\beta = E_m$ depends on the neutrino mass m_ν.
~~The density of states $\rho(E_m)$ is proportional to the neutrino momentum, i.e., to $(E_m - E_\beta)^{1/2}$~~

If the neutrino ~~has~~ possesses a finite mass
The beta distribution curve, then approaches the upper limit with a vertical tangent, insteads of a horizontal tangent. The recent experimental results show the β-spectrum ~~of screening to~~ with a vertical tangent near the upper limit and the neutrino mass ~~recent experimental results~~ → lies in the range $14\,eV < m(\bar{\nu}) < 46\,eV$

Selection rules in β-decay

$$I = L - S.$$

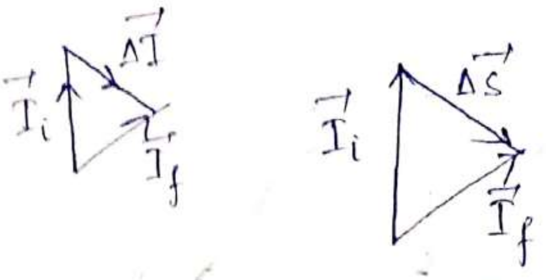
$$\Delta I = I_f - I_i$$

$$\Delta I = \Delta L + \Delta S$$

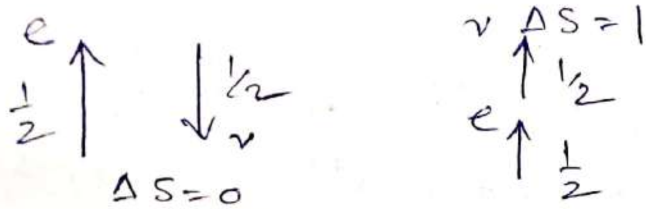
The orbital angular momentum carried away by the electron-neutrino pair must be equal to the change ΔL of the nucleus in β transition. So for allowed transition $\Delta L = 0$.

So for allowed transition

$$\Delta I = \Delta S$$



The electron and neutrino are spin $\frac{1}{2}$ particles. Their spins can be aligned either parallel or anti-parallel to each other, so that the total spin angular momentum carried away by them can be either $S = 0$ (anti-parallel) or $S = 1$ (~~anti~~ parallel).



In the first case, the spin change of the nuclear state will be $\Delta I = \Delta S = 0$.
Fermi selection rule.

In the second case $|\Delta I| = |\Delta S| = 1$

→ ~~Ca~~ Gamow-Teller selection rule.

For Gamow-Teller transition

$$\Delta I = 0, \pm 1.$$

In the G-T selection rule, the transition cannot take place from the state $I_i = 0$ to the state $I_f = 0$ because this transition involves the spin angular momentum change of one unit ($|\Delta S| = 1$).

Kurie plot

An allowed β-transition refers to the ~~dea~~ decay that brings about no change or a change by unity in the angular momentum quantum number, I, that is

either $\Delta I = 0$ ... Fermi's selection rule
or $\Delta I = \pm 1$ ... Gamow-Teller rule.

For such transitions, the probability of decay is a maximum and the number of particles emitted having momentum between p and p+dp, according to Fermi's theory, is

$$N(p)dp = AF(Z,p)p^2(W_m - W_k)^2 dp.$$

A = constant
W_k = kinetic energy of the β-particle
W_m = maximum value of E_k.
$F(Z,p)$ = Fermi function which depends on the force experienced by the emitted particle due to the nuclear charge.

Total energy E of the particle is given by

$$E = W_k + m_0c^2$$
$$\therefore dE = dW_k$$

$$E^2 = p^2c^2 + m_0^2c^4 \qquad 2E\,dE = c^2 p\,dp$$
$$p\,dp = \frac{E\,dE}{c^2}$$

maximum value of β-energy
$$E_m = W_m + m_0c^2$$

$$E_m - E = W_m - W_k$$

$$N(E)dE = N(p)dp = CF(Z,p)pE(W_m - W_k)^2 dE$$

$$\therefore N(W_k)\,dW_k = N(E)dE = CF(Z,p)p(W_k + m_0c^2)(W_m - W_k)^2 dW_k$$

c = constant.

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For low values of Z, $F(Z,p)$ = constant.

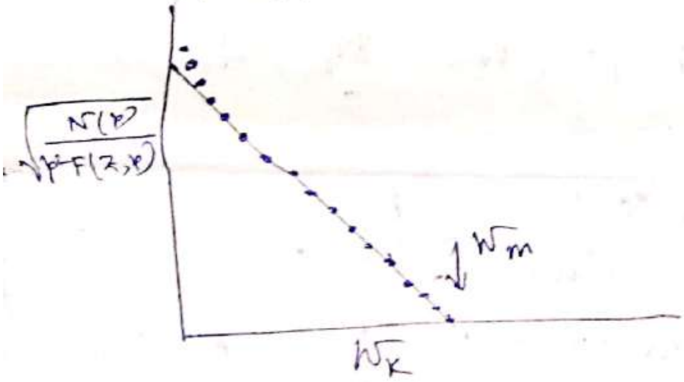
the ~~max~~ number of β-particles with zero kinetic energy ($W_K = 0$, $p = 0$) becomes zero.

For higher Z-values $F(Z,p) \neq$ constant.

and this results in a relatively higher number of low-energy β-particles

$$\sqrt{\frac{N(p)}{p^2 F(Z,p)}} = K(W_m - W_K)$$

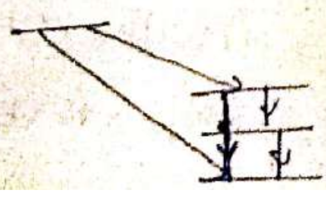
So the graph of $\sqrt{\frac{N}{p^2 F}}$ against W_K ~~is~~ would be a straight line. Such plots are called Kurie plot.



sometimes deviations are observed in straight line Kurie plots. This may be due to following two reasons:

① A forbidden transition will generally result in a concave-up type graph.

② Another reason for the deviation is a complex β-spectrum due to transitions to two or more states of the daughter nucleus.



nuclear states $\longrightarrow I^{\pi}$

For states with even L, parity even
odd L, " odd.

In allowed transitions $\Delta L = 0$
$L_i = L_f \rightarrow$ no change of parity

First forbidden transition
$\Delta L = 1$

a) Fermi ~~Secle~~ Selection rules
$\therefore\ \Delta S = 0,\quad \Delta I = 0, \pm 1$, yes.
$0 \rightarrow 0$ is forbidden

b) G-T. selection rule
$\Delta S = 1,\quad \Delta I = 0, \pm 1, \pm 2$, yes.

Transition	Fermi selection rule ΔI ($\Delta S = 0$)	Parity change	G.T. selection rule ($\Delta S = 1$) ΔI	Parity change
Allowed ($\Delta L = 0$)	0	No	$0, \pm 1$ (except $0 \leftrightarrow 0$)	No
First forbidden ($\Delta L = 1$)	$0, \pm 1$ (except $0 \rightarrow 0$)	Yes	$0, \pm 1, \pm 2$ (except $0 \rightarrow 0$, $\frac{1}{2} \rightarrow \frac{1}{2}$, $0 \rightarrow 1$)	Yes

Important point: ~~Par~~ Parity is not conserved in β-decay.

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Orbital electron capture

~~The~~ When a nucleus captures an orbital electron, only a neutrino is emitted, which is extremely difficult to detect. So, the only way of detecting an electron capture decay is to observe the characteristic x-rays of the daughter atom. When the electron is captured from K-shell, characteristic K x-rays are emitted, as the vacancy in the K-shell is filled up by the electrons from the outer L, M etc. shells. There is also an alternative process to the x-ray emission. The excess energy of the atom following the K-capture may be directly transferred to an L-electron which is thereby emitted. This is known as Auger electron and the process as Auger or radiationless transition.

Electron capture probability depends on the two factors: a) the probability of the electron being at the position of the nucleus and (b) the probability of capture of the electron by the nucleus. Since an electron in the ~~the~~ K shell has the largest probability of being at the position of the nucleus, so, K-capture has a greater probability to be observed compared to L-capture.

The ~~probabili~~ probability of K-capture depends on the nature of the electron-cloud around the nucleus. Thus the same nuclear transformation occurs with different probability.

Inverse β-decay

$p \rightarrow n + e^+ + \nu$

$p + e^- \rightarrow n + \nu$

The emission of a particle is equivalent to the absorption of its antiparticle. Since, the absorption of an antineutrino is equivalent to emission of a neutrino, the reaction

$$p + \bar{\nu} \rightarrow n + e^- \quad \text{—(1)}$$

involves the same physical process as that of

$$p \rightarrow n + e^+ + \nu$$

The reaction (1) is known as inverse β-decay. The probability of such reactions is vanishingly small since the cross-section for the interaction of neutrino or antineutrino with nuclei is about 10^{-46} to $10^{-45}\, m^2$.

The principal processes by which γ-rays interact with matter are (i) Photoelectric effect; (ii) Compton scattering and (iii) Electron-positron pair-production.

The total cross-sections for the above three processes

$$\sigma = \sigma_{ph} + \sigma_c + \sigma_p$$

In the first two processes, the γ-photons collide with the atomic electrons, while in the third, mainly with the nuclei. At lower energies, the first two processes are important. Pair production becomes important at higher energies ($E_\gamma > 3 MeV$).

(1) Photoelectric absorption: In the photoelectric process, ~~all the energy $h\nu$ of the incident photon~~ if a photon of energy $h\nu$ greater than the energy of binding B_e of an electron in an atomic shell, is incident on the latter, then the electron absorbs the whole quantum of energy of the photon and is emitted from the atom with a kinetic energy

$$E = h\nu - B_e$$

These electrons are known as photoelectrons. Since the γ-energy is usually quite high, ~~so~~ photoelectrons can be emitted from the inner electronic shells.

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The photoelectric absorption can take place from an electron bound in the atom. A free electron cannot absorb the entire energy of a photon, since energy and momentum conservation cannot be satisfied simultaneously. The more tightly bound the electron ~~as~~ is in an atomic shell, more probable is photoelectric emission. Thus the emission of the K-shell electrons is the most probable (80%).

The total photoelectric absorption cross-section for K-electrons is

$$\sigma_{ph} \propto Z^5$$

Hence high Z absorbers are more effective for ~~photoeled~~ photoelectron emission than low Z absorber.

At low photon energies, (~~below 500 keV~~ for Pb) the photoelectric effect gives the main contribution to the absorption.

② Compton scattering of γ-rays:

As the energy of the incident radiation is increased, the Compton scattering predominates over the photoelectric effect. In compton scattering, the incident photon is scattered by one of the atomic electrons and the latter is seperated ~~fo~~ from the atom. The photon moves off at an angle with its original direction with less energy than its original energy. The change of direction serves to remove the photon from the incident γ-ray beam.

σ_c more or less same for different elements.

(3) Pair production: For photons of high enough energy, a new process of absorption takes place and increases rapidly with increasing photon energy. In this process, when a photon of ~~suff~~ sufficiently high energy passes close to a nucleus in the absorber, it gets transformed into a pair of electrons, one carrying negative charge and the other ~~po~~ positive

$$\gamma \longrightarrow e^{+} + e^{-}$$

~~To~~ The energy of the photon must be at least twice the rest mass of the electron (1·02 MeV) ~~to~~ for producing electron- positron pair.

$$\sigma_p \propto Z^2.$$

pair-production probability is much greater in a high Z absorber.

Radiative Transitions

When a nucleus makes a transition from an excited state to a state of lower energy, it usually emits electromagnetic radiation (γ-ray) without change of A or Z. This is known as radiative transition.

The transition probability from a level is related to the width of the level. According to Heisenberg's uncertainty principle there is a limit to the sharpness of the energy of a quantum system which is related to its mean life time given by

$$\Delta E \, \Delta t \geqslant \hbar \qquad \hbar = \frac{h}{2\pi}, \; h \to \text{Planck's constant.}$$

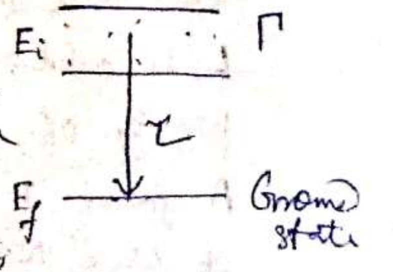
Unless the system has an infinite life-time its energy will always have an uncertainty $\Delta E = \Gamma$, which is called the width of the level. Greater the width, shorter is the life time τ of the level:

$$\tau = \frac{\hbar}{\Gamma}$$

The transition probability T per second which is equal to the decay constant λ is

$$T = \lambda = \frac{1}{\tau} = \frac{\Gamma}{\hbar}$$

So the width of the level determines the transition probability. Wider the level, more probable is the transition from it. Because of the finite width of the level, the emitted γ-ray line will have a finite spread in its frequency.



For a given γ-ray energy, photons can be emitted which differ in their angular momentum. The decay probability is the sum of the partial probabilities for the emission of these different types of γ-rays. ~~but in this sum one term, the first non-vanishing term, predominates~~. These radiations of distinct types from a nucleus are called multipole radiations. The partial probabilities is seen to depend very strongly on the angular momentum carried off by the photons. Photons can have only integral values of L. The multipole radiation is characterized by its order, given by 2^L. The value $L=0$ is ruled out as it corresponds to monopole. For $L=1$, the radiation corresponds to that emitted by a vibrating dipole; for $L=2$, it corresponds to the radiation from a vibrating quadrupole, for $L=3$ to that from an ~~octule~~ octupole, and so on. For each value of L there are two classes of radiation, called electric and ~~magma~~ magnetic multipole radiations, which differ in their parity. An electric multipole has even parity when L is even, and odd parity when L is odd. Magnetic multipole radiation has odd parity when L is even, and even parity whe L is odd; parity of electric multipole $= (-1)^L$; parity of magnetic multipole $= -(-1)^L$.

A given γ-decay process may leave the parity of the wave function of the nucleus unchange or there may be a change in the parity.

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Multipole type	Multipolarity (L)	Parity change
Electric dipole (E1)	1	Yes
Magnetic " (M1)	1	NO
Electric quadrupole (E2)	2	NO
Magnetic " (M2)	2	Yes
Electric ~~oct~~ octupole (E3)	3	Yes
Magneti " (M3)	3	NO
Electric 2^L pole (EL)	L	$(-1)^L$ [Yes of L odd, No for L even]
Magnetic 2^L pole (ML)	L	$(-1)^{L+1}$ [Yes for L even, No for L odd]

$I_i + I_f \geq L \geq |I_i - I_f|$

$I_i \longleftrightarrow I_f$	possible L-values	Parity change	Radiation
$0 \longleftrightarrow 0$	0	Yes, No	No radiation
$1 \longleftrightarrow 0$	1	Yes	E1
$2 \longleftrightarrow 0$	2	No Yes No	M1 M2 E2
$1 \longleftrightarrow 2$	1, 2, 3	Yes No	E1 (M2, E3) (M1, E2) M3
$1 \longleftrightarrow 3$	2, 3, 4	Yes No	(M2, E3) M4 E2 (M3, M4)
$2 \longleftrightarrow 3$	1, 2, 3, 4, 5	Yes No	E1 (M_2, E_3) (M_4, E5) (M1, E2) (M3, E4) M5

The probability of multipole emission decreases rapidly with increasing L. For the same L, emission of the electric radiation is much more probable than the emission of the magnetic radiation.

The transition probability for EL transitions is

$$\frac{1}{\tau_\gamma} \simeq \frac{1}{\lambda\!\!\!^{-}}\left[\frac{R}{\lambda\!\!\!^{-}}\right]^{2L}$$

and for ML transition is

$$\frac{1}{\tau_\gamma} \simeq \frac{1}{\lambda\!\!\!^{-}}\left[\frac{R}{\lambda\!\!\!^{-}}\right]^{2(L+1)}$$

$R \simeq 10^{-15} m$

$\lambda\!\!\!^{-} = \frac{\lambda}{2\pi} \sim 1.2\times10^{-10}$ to $1.2\times10^{-14} m$

$\therefore \frac{R}{\lambda\!\!\!^{-}} << 1$

The most allowed is the electric dipole (E1) transition. The next allowed are the electric quadrupole (E2) and magnetic dipole (M1) transitions.

Nuclear Models

To understand various properties of nucleus
↓
the nature of internucleon interactions to be known
But the exact mathematical form of this ~~force~~ interaction is yet to be known.
Various models have already been developed.
Liquid drop model, Fermi gas model, shell model.
However, none of the proposed theories gives a full understanding of the nature of the internucleon interaction.

Liquid drop model

Various macroscopic properties of the nucleus are similar to those of a liquid drop.
This model was first proposed by N. Bohr and F. Kalckar and was latter applied by Bethe and Weizsacker.

a) individual molecule within a ~~liqu~~ liquid drop exerts ~~at all~~ an attractive force upon a group of molecules in its immediate neighbourhood. This is known as the saturation of force.

~~If a~~ each molecule interacts with all molecules, the number of interacting pairs should be $N(N-1)/2 \sim \frac{N^2}{2}$.

~~on the~~ If ~~a~~ each molecule interacts with a limited number of molecules in its immediate vicinity, the number of interacting pairs would be linearly proportional to N so that the interaction potential should be proportional to N. This is experimentally supported by experimental evidences.

The total amount of heat required for evaporating a drop of liquid (latent heat) is linearly proportional to the number of molecules within the liquid. Hence the heat required per molecule is constant.

Binding energy $E_B \propto A$

Binding fraction $f_B = \frac{E_B}{A} \simeq$ constant for most nuclei.

Thus we can say the internucleon force within a nucleus attains a saturation value, so that each nucleon can interact only with a limited number of nucleons in its close vicinity.

b) The attractive force near the nuclear surface is similar to the force of surface tension on the surface of the liquid drop.

c) Like liquid drop, the density of the nuclear matter is independent of its volume.

$$R \propto A^{1/3}$$

$$V \propto A$$

$$M \sim A$$

$$\rho_m = \frac{M}{V} \sim \text{constant.}$$

$\therefore \rho_m$ is independent of A.

d) emission of neutrons, protons, deuterons in nuclear reactions
→ evaporation of liquid drop.

e) the internal energy of a ~~liquid drop~~ nucleus
→ heat ~~of a~~ energy within a liquid drop

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f) Formation of a short lived compound nucleus → process of condensation from the vapour to the liquid phase.

The liquid drop model is not very successfull in explaining the low lying excited states.

Bethe-Weizsacker Formula

$M(A,Z) \longrightarrow$ atomic mass of the isotope of an element X

$$M(A,Z) = ZM_H + NM_n - E_B$$

The binding energy E_B is expressed as a sum of several terms.

a) Volume energy

$E_B \propto A$

$\cancel{E_B \propto A}$

$E_B = a_1 A$

$\because V \propto A$ This term is called volume energy

b) Surface energy

Since the nucleus is assumed to resemble a spherical liquid drop of radius $R = r_0 A^{1/3}$, a force similar to the surface ~~energy~~ tension of a liquid acts on the nucleon near the free surface of the nuclear surface.

The nucleons ~~at the~~ near the free surface are acted upon by the attractive forces due to the nucleons inside the ~~surface~~ sphere. there are no forces acting from the outside. As a result the nuclear assumes a spherical shape.

This surface force is proportional to the surface area $4\pi R^2 = 4\pi r_0^2 A^{2/3}$

This surface force reduce the binding energy.

$$E_S = -a_S A^{2/3}$$

(iii) Coulomb Energy

~~This a~~ arises ~~do~~ due to Coulomb repulsion between the protons.

weakens the binding energy.

Nuclear charge $Q = Ze$

charge density $\rho_c = \frac{Q}{\frac{4}{3}\pi R^3} = \frac{3Ze}{4\pi r_0^3 A}$

The charge within a sphere of radius r is

$q = \frac{4}{3}\pi r^3 \rho_c$

$dq = 4\pi r^2 \rho_c dr$

Total workdone in bringing dq from infinity to the radius r against the field of the charged sphere

$$dW = \frac{1}{4\pi\varepsilon_0} \frac{q\, dq}{r}$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{4}{3}\pi r^3 \rho_c \cdot \frac{4\pi r^{\cancel{2}} \rho_c dr}{\cancel{r}}$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{16\pi^2}{3} \rho_c^2 r^4 dr$$

Hence total work done in forming the total sphere of radius R

$$W = \frac{1}{4\pi\varepsilon_0} \frac{16\pi^2}{3} \rho_c^2 \int_0^R r^4 dr$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{16\pi^2}{3} \rho_c^2 \frac{R^5}{5}$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{\cancel{16\pi^2}}{\cancel{3}} \frac{Q^2}{\frac{16}{9}\cancel{\pi^2} R^6} \frac{R^5}{5}$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{3}{5} \frac{Q^2}{R}$$

The potential energy is the -ve of the total work done. So Coulomb energy is

$$E_c = -\frac{3}{5}\frac{Q^2}{4\pi\varepsilon_0 R} = -\frac{3}{5}\frac{(Ze)^2}{4\pi\varepsilon_0 R}$$

$$= -\frac{3}{5}\frac{(Ze)^2}{4\pi\varepsilon_0 r_0 A^{1/3}} = -a_3\frac{Z^2}{A^{1/3}}$$

$$a_3 = \frac{3}{5}\frac{e^2}{4\pi\varepsilon_0 r_0}$$

This is not exact,
Corrections are needed due to
(i) non-uniformity of the charge distribution
(ii) ~~the~~ discrete arrangement of the charges on the protons
(iii) effect of uncertainty in the localization of protons
(iv) non-sphericity of the nucleus.
(v) corrections of the position of the protons.

(iv) <u>Asymmetry energy</u>

For lighter nuclei $N \simeq Z$

But for heavier nuclei Coulomb repulsion among the protons ~~are notes~~ is not ignorable which disturbs the nuclear stability. So ~~neutron~~ extra neutrons are present to provide additional n-n bonds.
So ~~light~~ for heavier nuclei $N > Z$.
This asymmetry weakens the binding energy, which is proportional to $(N-Z)^2$
$\sim (A-2Z)^2$

$$\therefore E_a = -a_4\frac{(A-2Z)^2}{A}$$

(v)

Pairing Energy

For even A

e-e strongly bound compared to o-o

B.E of e-o and o-e are intermediate between the ~~e~~ e-e and o-o nuclei.

Binding energy

e-e (even A) > e-o or o-e (odd A) > o-o (even A)

This is due to pairing of same type of nucleons with opposite spins.

Such pairing increases the strength of the binding energy.

This is maximum for e-e where all the nucleons form pairs.

In odd A nuclei there is one unpaired nucleon e-o → neutron
o-e → proton
↓
weakens the binding by about 2 to 3 MeV

In o-o nuclei there are two unpaired nucleons, one proton and one neutron.

The binding of such nuclei are further weakened by about 2 to 3 MeV.

The pairing energy term

$\delta(A, Z)$ depends on A only

$\delta = a_5 A^{-3/4}$ $\quad$ $\delta \to 0$ for odd A
$\to$ +ve for e-e
$\to$ -ve for o-o

$E_B = E_V + E_S + E_C + E_a + \delta$

$\delta = 0 \to$ odd A
added for e-e A
subtracted for o-o A

Mass Parabola: Stability of Nucleus against β.

$$M(A,Z) = Z M_H + N M_n - a_1 A + a_2 A^{2/3} + a_3 \frac{Z^2}{A^{1/3}} + a_4 \frac{(A-2Z)^2}{A}$$

$= f_A + pZ + qZ^2$ → parabola for a given A. (neglecting δ)

$$f_A = A(M_n - a_1 + a_4) + a_2 A^{2/3}$$

$$p = -4a_4 - (M_n - M_H)$$

$$q = \frac{1}{A}(a_3 A^{2/3} + 4a_4)$$

Diff. M. w. r. to Z

$$\left.\frac{\partial M}{\partial Z}\right|_A = p + 2qZ$$

At the lowest point

$$\left.\frac{\partial M}{\partial Z}\right|_A = 0$$

or $$Z_A = -\frac{p}{2q} = \frac{(M_n - M_H + 4a_4)A}{2(a_3 A^{2/3} + 4a_4)}$$

$M(A,Z) = Z M_H + N M_n - B.E.$
$= Z M_H + (A-Z) M_n - B.E$
$= Z(M_H - M_n) + A M_n - B.E.$
$\because M_H \simeq M_n$
$M(A,Z) = A M_n - B.E.$

$$M(A, Z_A) = f_A + pZ_A + qZ_A^2$$

$$M(A,Z) - M(A,Z_A) = p(Z - Z_A) + q(Z^2 - Z_A^2)$$
$$= (Z - Z_A)[p + qZ + qZ_A]$$
$$= (Z - Z_A)\left[p + qZ + q\cdot\left(-\frac{p}{2q}\right)\right]$$
$$= (Z - Z_A)\left[\frac{p}{2} + qZ\right]$$
$$= q(Z - Z_A)\left(Z + \frac{p}{2q}\right)$$
$$= q(Z - Z_A)(Z - Z_A) = q(Z - Z_A)^2$$
$= +ve$

Hence, the mass parabola for a given isobar has lowest point at $Z = Z_A$.

Since M(A,Z) has lowest value at $Z = Z_A$ for a given A, this nucleus will have the largest binding energy amongst the isobars for a given A, i.e. Z_A will give the value of Z for most stable isobar.

Putting the numerical values of M_n, M_H, a_3 and a_4,

$$Z_A = \frac{A}{1.98 + 0.015 A^{2/3}}$$ → not an integral value usually.

The value of Z nearest to Z_A corresponds to the actual stable nucleus. for a given A.

Ex $A = 63$, $Z_A = 28.4$

$^{63}Cu \to Z = 29 \to$ stable ~~nuclei~~.

$A = 109$,

$Z_A = 46.94$

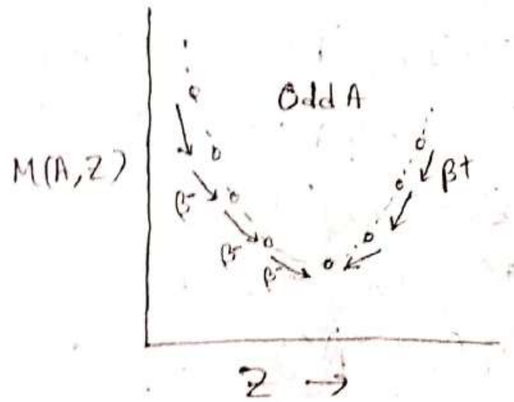
$^{109}Ag \to Z = 47 \to$ stable.

If the pairing energy δ is taken into account, the mass parabola falls into two categories

① odd-A

$\delta = 0$.

It gives single parabola for a given A.



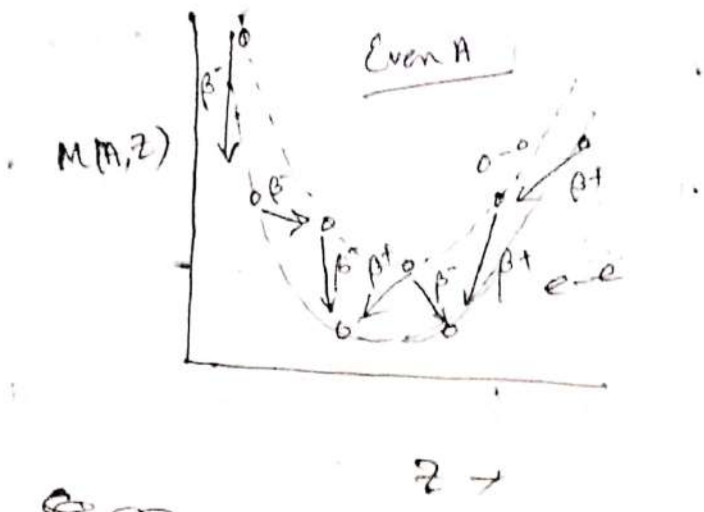
② Even-A

For e-e nuclei δ is subtracted

and for o-o nuclei δ is added to $M(A,Z)$

So, There are two parabolas for the same mass numbers.

The parabola for the odd-odd nuclei lies above the ~~even~~ that for the even-even nuclei. for the same A by an amount 2δ.



~~Since~~

The odd Z - even A nuclides lie on the upper curve and are thus unstable with respect to those on lower one. So, no stable odd Z - even A nucleides should exist. The only exceptions are ^2H, ^6Li, ^{10}B and ^{14}N which are light ~~nucl~~ ~~nuclei~~ nuclei that do not come under liquid drop model.

For even Z - even A nucleides, two isobars with Z differing by 2 units can lie close to the bottom of the lower curve. They constitute stable isobaric pairs. (ex. ^{64}Ni, ^{64}Zn).

The o-o isobar between these stable isobaric pairs falls on the bottom of the upper parabola and has an atomic mass greater than those of either of the above two. Hence, this particular o-o isobar is not stable. It shows both β^+ and β^- activities. (ex. ^{40}K, ^{64}Cu etc.)

(* *) All isobars having B.E. less than the most stable one will lie on the two arms of the parabola. Their masses will be greater than that of the stable isobar. ~~and they will decay by~~ The isobars to the left of the stable one decay by β^- emission as they have fewer protons than the stable one, while those to the right having an excess proton decay by β^+ emission or K-capture, or by both.

~~Nuclear~~

Nuclear shell structure.

Atomic electrons shell structure

$n = 1 \quad 2 \quad 3 \quad 4$

K L M N

~~Each~~ Each of them possesses sub shells

$l = 0 \quad 1 \quad 2 \quad 3$

s p d f

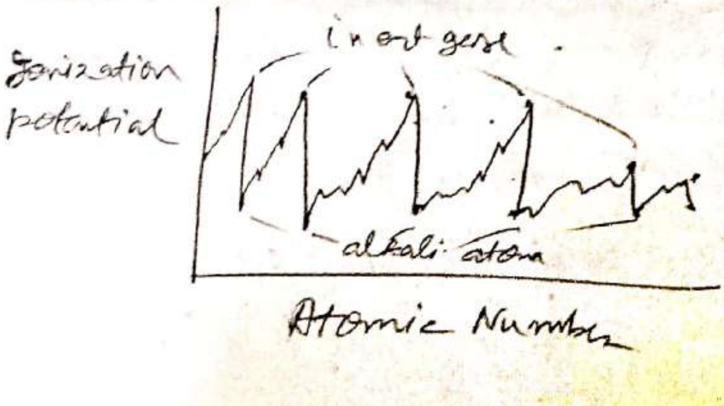
max elect. 2 6 10 14

In inert all gases except He, the p-sub shell is completely filled.

In He s-subshell is completely filled.

In all these elements electrons are tightly bound, their first ionization potential is relatively high.

Alkali atoms immediately follows the inert gases. There is one electron in s subshell just outside the inert gas core. This electron is very weakly bound. ~~There is a sud~~ The first ionization potentials for alkali atoms are relatively low compared to inert gases.



Nuclei ~~with~~ containing certain numbers of protons or neutrons exhibite very high stability.

P	2	8	20	28	50	82	
N	2	8	20	28	50	82	126

magic numbers.

Semi magic number N or Z = 40.

Nuclei containing magic numbers of protons and neutrons both show exceptionally high stability. They are called doubly magic.
^{4}He (Z=2, N=2), ^{16}O (Z=8, N=8), ^{40}Ca (Z=20, N=20), ^{48}Ca (Z=20, N=28), ^{208}Pb (Z=82, N=126).

Evidence for existence of shell structure

a) Nuclei containing magic numbers of protons and ~~also~~ neutrons show very high stability, compared to the nuclei containing one more nucleon of the same kind.
Separation energy of a neutron from a nucleus containing a magic number of neutrons is large compared to that for a nucleus containing one more neutron. Similar is the case for a proton.

b) The naturally occuring isotopes, whose nuclei containg magic numbers of neutrons or protons, have generally greater relative ~~an~~ abundances (>60%).

c) The number of stable isotopes of an element containing a magic number of proton is usually large compared to those for other elements.

Ca → Z = 20 → s.i → 6
Ar → Z = 18 → 5
Ti → Z = 22 → 5

d) The number of naturally occurring isotones with magic ~~and~~ numbers of neutrons is ~~usulal~~ usually large compared to those in the immediate neighbourhood.

$N = 82 \rightarrow$ stable isotone $\rightarrow 7$

$N = 80 \rightarrow 3$

$N = 84 \rightarrow 2$

e) The stable ~~ck~~ end products of all the three ~~and~~ natural radioactive series are the three isotopes of lead having $Z = 82$ in all of them.

f) Nuclei with magic numbers of neutrons or protons have their first excited states at higher energies than in the cases of the neighbouring nuclei.

g) The neutron capture cross-sections of the nuclei with magic number of neutrons are ~~usu~~ usually low.

h) ~~The of~~ α-disintegration energy for a given Z and different A shows sudden discontinuity at $N = 126$.

i) Similar discontinuities are observed amongst the β-emitters at the magic neutron or proton numbers.

Nuclear Shell Model

$l=4 \cdot 2n$ 1,2 2,0 $l=2 \cdot 2n$ 1,0 $l=3 \cdot 2n$ 1,1

~~In the shell model of the nucleus, each nucleon in the nucleus is supposed to move independently of the ~~oth~~ others in the field representing the time averaged effect of the interactions of all the other nucleons. Unlike in the atomic case, no central ~~an~~ agency is known to exist in the nucleus.~~

The nuclear shell model assumes that each nucleon stays in a well defined quantum state. But, unlike the atom, no central agency is known to exist in the nucleus, i.e., the nucleus has no fixed centre of charge. In this model, each nucleon in the nucleus is supposed to move independently of the others in the time-averaged field of all other nucleons, and remains confined to its own orbit unperturbed by others. Each nucleon with its spin $\frac{1}{2}\hbar$ behaves like an independent particle, subject to the additional condition, imposed by Pauli's exclusion principle that no two identical nucleons can be in the same quantum state. In terms of Schrödinger's eqn., each nucleon thus moves in the same potential which may be taken as spherical in the simplest case. ~~In this~~ ~~Further, in this independent particle model, it~~ according to Mayer, Jensen and others suggestion, it is assumed that there is a strong spin-orbit coupling for each individual nucleon.

Hence each nucleon energy level with a given value of l for its orbital quantum number splits into two sub-levels with the total angular quantum number j having the value $j=l+\frac{1}{2}$ and $j=l-\frac{1}{2}$, except when $l=0$, in ~~the~~ which case j takes only one value, i.e. $j=\frac{1}{2}$.

The spin-orbit splitting of the two levels is proportional to $(l+\frac{1}{2})$ given by

$$E_{(l+\frac{1}{2})} - E_{(l-\frac{1}{2})} \quad \Delta E_{ls} = (l+\tfrac{1}{2}) \langle f(r) \rangle \approx 10(2l+1)A^{-2/3}\,\text{MeV}$$

where $\langle f(r) \rangle$ is the average value of the contains the average over radial part of the spin-orbit potential. Since, ΔE_{ls} is +ve, it is obvious that the state with $j = l+\frac{1}{2}$ lies below the state $j = l-\frac{1}{2}$. Hence the state $j = l+\frac{1}{2}$ sub-level has a lower energy than $j = l-\frac{1}{2}$ sub-level, and the former gives a more tightly bound nucleonic state. The separation between two sub-levels with a given l is large and increases rapidly with l. With $l \geq 4$, the sub-levels lie in the different shells. The total angular momentum quantum number of the nucleus is obtained by $\sum j$, i.e., adding the j-values of individual nucleons, subject however to usual quantum conditions.

The sequence of the energy levels, taking spin-orbit interaction into account is shown below, where n is the radial quantum number, l is the azimuthal quantum number, j is the total angular quantum number. ~~and~~ n and l are related to the principal quantum number λ by the relationship

$$\lambda = 2n + l - 2$$

where $n = 1, 2, 3,$

$l = 0, 1, 2, \ldots$

and $\lambda = 0, 1, 2, 3, \ldots$

In accordance with Pauli's exclusion

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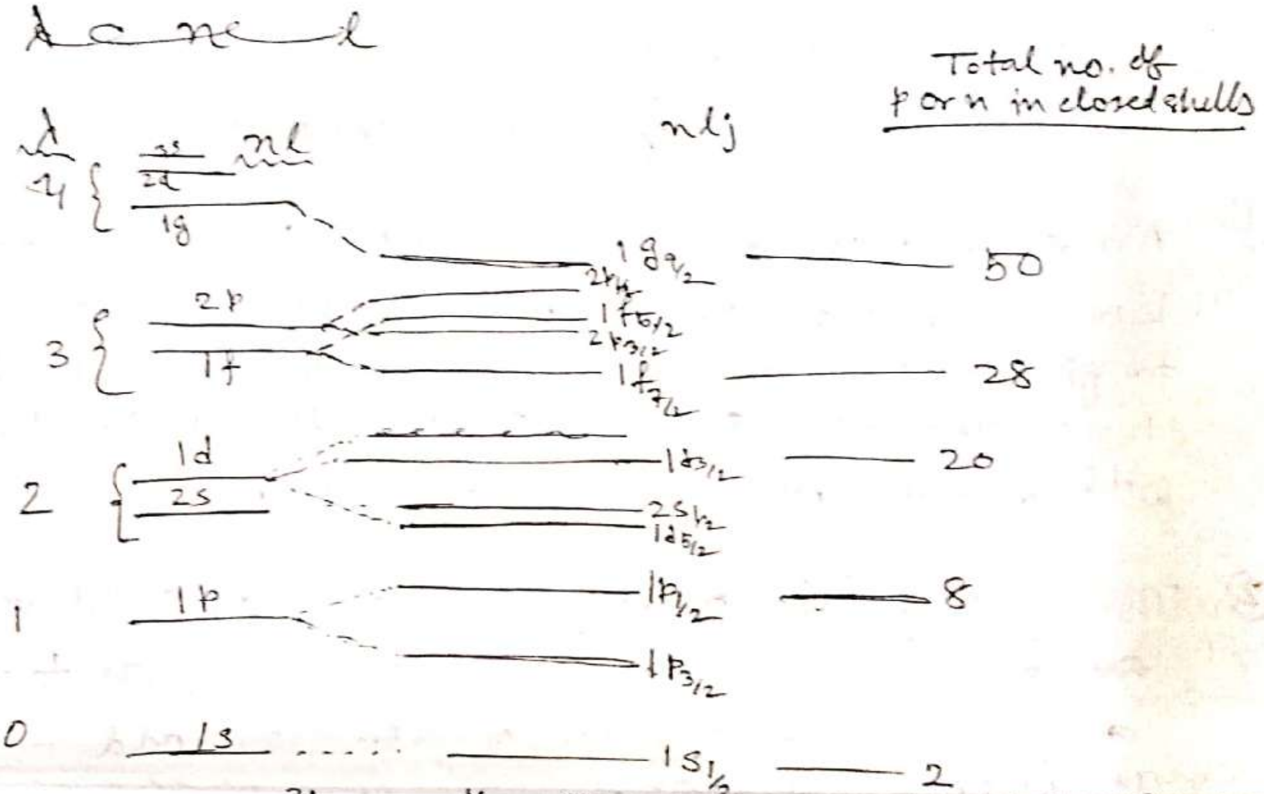
principle, each sublevel of a given j can accommodate a maximum of $(2j+1)$ nucleons of either kind for which the magnetic quantum numbers m_j are different.



$$(1s_{1/2})^2 \,\Big|\, (1p_{3/2})^4 (1p_{1/2})^2 \,\Big|\, (1d_{5/2})^6 (2s_{1/2})^2 (1d_{3/2})^4 \,\Big|\, (1f_{7/2})^8 \,\Big|\, (2p_{3/2})^4 (1f_{5/2})^6 (2p_{1/2})^2 (1g_{9/2})^{10} \,\Big|\, \ldots$$

Apart from the magic numbers, the shell model ~~can~~ can successfully predict ~~successfully~~ spin, magnetic moment ~~of~~ in ground states, spin of excited states, islands of isomerism. However, it fails in explaining the observed large electric quadrupole moments of the nuclei far from closed shells.

Nuclear Spin

s p p d s d f p f p g

Single nucleon $1s_{1/2} | 1p_{3/2}\ 1p_{1/2} | 1d_{5/2}\ 2s_{1/2}\ 1d_{3/2} | 1f_{7/2} | 2p_{3/2}\ 1f_{5/2}\ 2p_{1/2}\ 1g_{9/2} |$

↓

total angular momentum $j = l + s$

Total angular momentum I of the system of nucleons

$$I = \sum j_i \rightarrow j\text{-}j \text{ coupling.}$$

① An even number of nucleons of any one kind in the same state j always combines to give the ~~&~~ resultant spin 0 and even parity. Hence, the total angular momentum of any shell with an even number of nucleons is zero.

② The ~~&~~ angular momentum of any shell with an odd number of nucleons is equal to the angular momentum of the last odd unpaired nucleon.

③ The neutron and proton states fill independently

Once the basis of above assumptions,

* For N or Z even: I=0

① ~~For N even, Z odd~~ Hence, for

(A) Even A

e-e ⟶ $I = 0, \pi = +$ $\quad I^\pi = 0^+$

(B) Odd-A

(i) For o-e i.e. for N=even, Z=odd

I will depend on last unpaired proton

(ii) For e-o, i.e. for N=odd, Z=even,

I will depend on last unpaired neutron.

(C) For o-o nuclei, I depends on both last ~~&~~ unpaired neutron and last unpaired proton and is determined by Nordheim rule.

Nordheim Rule

~~The partial~~ ~~partially filled neutron~~

The last unpaired neutron is characterized by (N_n, l_n, j_n)

The last unpaired proton is characterized by (N_p, l_p, j_p)

$I \to |j_p - j_n| \cdots (j_p + j_n)$

$\pi = (\pi)^p \times (\pi)^n$

Strong rule

(i) if $j_p - l_p + j_n - l_n = 0$, $I = |j_n - j_p|$

~~$\pi = (\pi)^p \times (\pi)^n$~~

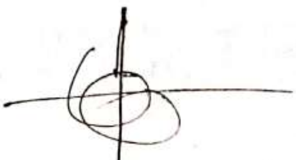
Weak rule

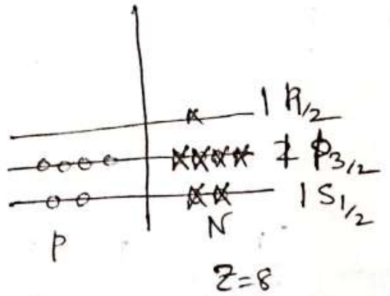
(ii) if $j_p - l_p + j_n - l_n = \pm 1$

$I = |j_n - j_p|$ or $(j_n + j_p)$

Ex

① $^{13}_{6}C$ $Z = 6$, $N = 7$



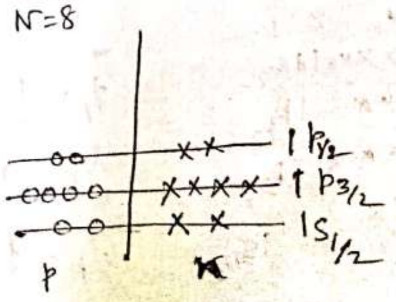


Due to unpaired neutron $I = \frac{1}{2}$

$\pi = (-1)^1 = -1$

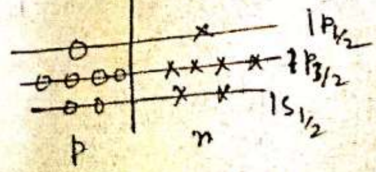
$I^\pi = \frac{1}{2}^-$

② $^{16}_{8}O$ $Z=8$, $N=8$



$I^\pi = 0^+$

③ $^{14}_{7}N_7$



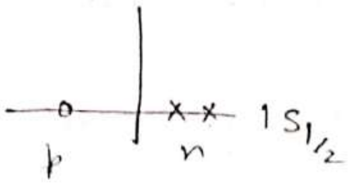
Ang. momentum of ~~Due to~~ unpaired proton $j_p = \frac{1}{2}$ with $l_p = 1$ and that of unpaired neutron $j_n = \frac{1}{2}$, $l_n = 1$

$j_p - l_p + j_n - l_n = \frac{1}{2} - 1 + \frac{1}{2} - 1$
$= -1$

Hence, by Nordheim's rule $I = |j_p - j_n|$ or $(j_p + j_n)$
$= 0$ or 1

$I^\pi = 0^+$ or 1^+

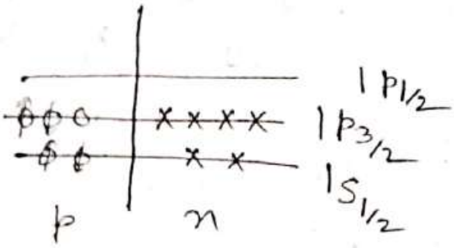
(4) $^{3}_{1}H$ $Z = 1$ $N = 2$



$I = \frac{1}{2}$ $\pi = +$

$I^{\pi} = \frac{1}{2}^{+}$

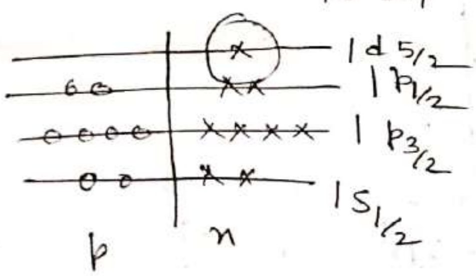
(5) $^{11}_{5}B$ $Z = 5$ $N = 6$



$I = \frac{3}{2}$ $\pi = -$

$I^{\pi} = \frac{3}{2}^{-}$

(6) $^{17}_{8}O_{9}$ $Z = 8$ $N = 9$



$I = \frac{5}{2}$

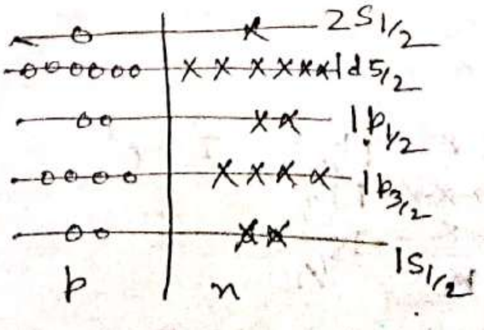
$\pi = +$

$I^{\pi} = \frac{5}{2}^{+}$

(7) $^{30}_{15}P$ $Z = 15$ $N = 15$

$\pi_p = +$ $\pi = (+) \times (+)$

$\pi_n = +$ $= +$



$j_p = \frac{1}{2}$ $j_n = \frac{1}{2}$

$l_p = 0$ $l_n = 0$

$j_p - l_p + j_n - l_n$

$= \frac{1}{2} - 0 + \frac{1}{2} - 0$

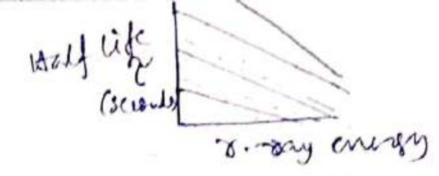
$= 1$

$I = |j_p - j_n|$ or $(j_p + j_n)$

$= 0$ or 1

$I^{\pi} = 0^{+}$ or 1^{+}

Nuclear Isomerism



For high multipolarity (large L) and low energy(E) the radiative transition half-lives are long ~~and hence~~ enough to be measurable. Although most γ-ray lifetimes have ~~to~~ been found to be very short, more than 250 cases are known in which the lifetime for γ-decay is measurable, with observed half-lives from 10^{-10} sec. to many years. Hence the probabilities of such transition is low. [Such long lived excited nuclear states are known as isomers and these delayed transitions are called isomeric transitions] Because of these delayed transitions, pairs of nuclear species exist which have the same atomic and mass numbers, i.e., they are isotopic and isobaric, but have different radioactive properties; these nuclides are called ~~as~~ nuclear isomers and their existence is referred to as nuclear isomerism.

Internal pair creation

If the energy of ~~radiative~~ transition between two nuclear states exceeds $2m_ec^2 = 1.02$ MeV, then sometimes a pair of electron and positron is produced, instead of a radiative transition between the two states. This is known as internal pair creation.