→ Considering a particular configuration of elastic collision of two bodies (A and B) having identical mass (m) the Newtonian conclusion or result is $\vec{u}_{yA} = \vec{u}_{yB}$; where u (small letter) refers to the velocity before the collision. After the collision, it is represented by U (capital letter). The analysis is regarding an elastic collision between two identical bodies as observed by different inertial observers i.e S and S' - Newtonian Mechanics.

→ What about "such collision" with the Lorentz transformations? i.e contradict the relativistic velocity transformations.

Soln: from Newtonian results in S-frame i.e $u_{yA} = u_{yB}$. — (1) [y-components are not affected]

for body B, in S'-frame w.r.t S-frame; i.e $u_{yB'} = \dfrac{u_{yB}\sqrt{1-\beta^2}}{1-\frac{u_{xB}v}{c^2}}$ (2) (for velocity in the x-axis i.e u_{xB})

" " A, " for which $u_{xA} = 0$; i.e $u_{yA'} = u_{yA}\sqrt{1-\beta^2}$. —(3)

✓ from $u_{yB'}$ and $u_{yA'}$ it is clear that the y-components of velocity are affected or "gets influenced" by the relativistic transformations.

✓ from (1), (2) and (3); velocities are equal in S-frame but $\underbrace{\text{not equal in } S'\text{-frame}}_{u_{yB'} \text{ and } u_{yA'}}$.

In contrast to eqn (1), for Lorentz transformation $u_{yB'} = u_{yA'}$ if using (2) & (3)

$$u_{yA} = u_{yB}\frac{1}{1-\frac{u_{xB}v}{c^2}} \quad \text{— (4). [y-component gets affected].}$$

→ from Classical formulas, $\vec{p} = m\vec{u}$ and $\vec{p}' = m\vec{u}'$

→ The Newtonian formulation of the momentum conservation law breaks down for velocities, $u_{xB} \to c$ and $v \to c$. Eqn (4) reduces to Newtonial formulation when $u_{xB} \ll c$ and $v \ll c$.

✓ RELATIVISTIC MOMENTUM :-

✓ If we have $2m_A u_{yA} = 2m_B u_{yB}$ (i.e unlike before /above stated as 2 identical bodies)

then $m_B = m_A \dfrac{u_{yA}}{u_{yB}} = \dfrac{m_A}{1-u_{xB}v/c^2}$ → In S-frame → Relativistic masses, m_A and m_B are not equal. When $v = u_{xB'}$ and $u_{xB'}$ related to u_{xB} by the Lorentz velocity transformation

$$u_{xB'} (= v) = \frac{u_{xB} - v}{1 - u_{xB}v/c^2} \quad \text{Solving, } v = \frac{c^2}{u_{xB}}\left[1 - \sqrt{1-(u_{xB}/c)^2}\right].$$

Next, $m_B = \dfrac{m_A}{\sqrt{1-(u_{xB}/c)^2}}$.

S' frame → observer sees 2 bodies moving past each other making a grazing collision

S frame → " " body A (at rest) & body B " " " " " " " with speed u_{xB}.

∴ In frame S, body A is at rest and so mass → m_A (Newtonian mass) ∴ Rest mass m_0;
" " ", body B is moving with speed u_{xB} → mass is m_B and velocity u.

So, $m = \dfrac{m_0}{\sqrt{1-u^2/c^2}}$ → ∴ As $u = 0$ or $u \ll c$ i.e $\frac{u}{c} \to 0$ we have $m \to m_0$.

In conclusion, the conservation of momentum in collision a law that is experimentally valid in all reference frames the momentum is defined not as $m_0\vec{u}$, but as

$\vec{p} = \dfrac{m_0\vec{u}}{\sqrt{1-u^2/c^2}}$; where $p_x = \dfrac{m_0 u_x}{\sqrt{1-u^2/c^2}}$, $p_y = \dfrac{m_0 u_y}{\sqrt{1-u^2/c^2}}$ and $p_z = \dfrac{m_0 u_z}{\sqrt{1-u^2/c^2}}$.

Q: From the above eqn $m = \dfrac{m_0}{\sqrt{1-u^2/c^2}}$; $m > m_0$ or $f = \dfrac{m-m_0}{m_0} = \dfrac{m}{m_0} - 1 = \dfrac{1}{\sqrt{1-\beta^2}} - 1$

or $\boxed{\beta = \dfrac{\sqrt{f(2+f)}}{(1+f)}}$. ∴ $\beta(f) \to$

f	0.001 or 0.1%	0.01	0.1	1 or 100%	10	100
β	0.045	0.14	0.42	0.87	0.994	0.995

Rest mass, m_0 and Relativistic mass, m.

→ The Relativistic Force Law & the Dynamics of a Single particle:-
In Relativistic mechanics, Newton's 2nd law :- $\vec{F} = \frac{d}{dt}(\vec{p}) = \frac{d}{dt}\left(\frac{m_0\vec{u}}{\sqrt{1-u^2/c^2}}\right)$.
From the law of conservation of relativistic momentum, $\vec{F} = 0 \Rightarrow \vec{p} = (m_0\vec{u})\gamma$ is a constant.
Q ∴ In absence of any external forces, the momentum is conserved.

If $\vec{F} \neq 0$, then for a system of interacting particles, the total relativistic momentum changes by an amount $\Delta\vec{P} \equiv \int \vec{F}\,dt$ ⇒ The total IMPULSE given to the system. (Q44) R. Resnick P155

For high-speed charged particles, the motion can be described as $\underbrace{q(\vec{E}+\vec{u}\times\vec{B})}_{\text{Lorentz e-m. force}} = \frac{d}{dt}\left(\frac{m_0\vec{u}}{\sqrt{1-u^2/c^2}}\right)$ → all measured in the same frame of reference.

Single particle & system of particles. or many-particle systems.

Newtonian mechanics, kinetic energy of a particle $K = \int_{u=0}^{u=u} \vec{F}\cdot d\vec{l}$, where $\vec{F}\cdot d\vec{l}$ is the work done by the external force $\vec{F}$ in displacing the particle through $d\vec{l}$.
Considering 1-d motion i.e say x ∴ $K = \int_{u=0}^{u=u} \vec{F}\,dx = \int m_0\left(\frac{du}{dt}\right)dx = \int m_0\,du\,\frac{dx}{dt}$.

or $K = m_0 \int_0^u u\,du = \frac{1}{2}m_0u^2$. [Newtonian mechanics, mass m is constant and not varying with v ∴ Force $m_0 a = m_0\left(\frac{du}{dt}\right)$

For (ii)

Relativistic mechanics, redefining, $K = \int_{u=0}^{u=u} F\,dx = \int \frac{d}{dt}(mu)\,dx = \int d(mu)\,\frac{dx}{dt}$.

or $K = \int_{u=0}^{u=u} \vec{F}\,dx = \int \frac{d}{dt}(mu)\,dx = \int d(mu)\frac{dx}{dt} = \int (m\,du + u\,dm)\,u = \int_{u=0}^{u=u}(mu\,du + u^2\,dm)$.

Here, both m and u are variables. & $m = m_0/\sqrt{1-u^2/c^2}$.

$\therefore m^2c^2 - m^2u^2 = m_0^2c^2$. Taking differentials, we get

$2mc^2dm - m^2 2u\,du - u^2 2m\,dm = 0$. or $mu\,du + u^2dm = c^2dm$ (dividing by $2m$)

$\therefore K = \int_{u=0}^{u=0} c^2dm = c^2\int_{m=m_0}^{m=m} dm = \underline{mc^2 - m_0c^2}$ using $m = \dfrac{m_0}{\sqrt{1-u^2/c^2}}$

we get $K = m_0c^2\left[\dfrac{1}{\sqrt{1-u^2/c^2}} - 1\right]$. where $E = mc^2$ is the total energy of the particle

$\boxed{\text{The Rest Energy}}$ → Energy of the particle at rest, when $u=0$ and $K=0$. → $\boxed{E_0 = m_0c^2}$

$\therefore$ $\boxed{E = m_0c^2 + K.}$ K is the kinetic energy of the particle.

From, $K = m_0c^2[(1/\sqrt{1-u^2/c^2}) - 1] = m_0c^2\left[\left(1-\frac{u^2}{c^2}\right)^{-1/2} - 1\right]$ using Binomial thm expansion in u/c. $\therefore$ $K = m_0c^2\left[1 + \frac{1}{2}\left(\frac{u}{c}\right)^2 + \frac{3}{8}\left(\frac{u}{c}\right)^4 + \cdots - 1\right] = \frac{1}{2}m_0u^2$.

i.e $u/c \ll 1$ i.e Newtonian limit of the relativistic result.

As $u \to c$, $K \to \infty$; from $K = \int_{u=0}^{u=u} F.dx = \int_{u=0}^{u=u}(mu\,du + u^2dm)$ → an infinite amount of work would be required to be done on the particle to accelerate it upto the speed of light.

From $K = (m - m_0)c^2$, a change in the kinetic energy of a particle is related to a change in its (inertial) mass.

Connection bet^n. the kinetic energy K of a rapidly moving particle and its momentum p.

(p123) Consider momentum $\vec{p} = \dfrac{m_0\vec{u}}{\sqrt{1-u^2/c^2}}$ and (K.E) $K = m_0c^2\left[\dfrac{1}{\sqrt{1-u^2/c^2}} - 1\right]$.

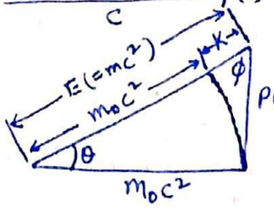
$p^2\left(1 - \frac{u^2}{c^2}\right) = m_0^2u^2$.

Q: Useful relations bet^n p, E, K and m_0 for relativistic particles:

(1) $K = c\sqrt{m_0^2c^2 + p^2} - m_0c^2$; (2) $p = \dfrac{\sqrt{K^2 + 2m_0c^2K}}{c}$; (3) $m_0 = \dfrac{\sqrt{E^2 - p^2c^2}}{c^2}$

$(K + m_0c^2)^2 = E^2 = (pc)^2 + (m_0c^2)^2$

→ Relation bet^n total energy, E; rest energy m_0c^2; and momentum p.



[Pythagorean relation]

It can also be shown that $\sin\theta = \beta = v/c$ and $\sin\phi = \sqrt{1-\beta^2}$.

from (2) it can be shown that $p = \sqrt{2m_0K}$. when $K^2 \ll 2m_0c^2K$.

High-energy physics (HEP) → To estimate the total energy of a particle when its momentum is given or vice versa. i.e Diff. w.r.t p of the eq^n $E = c\sqrt{p^2 + m_0^2c^2}$.

$\dfrac{dE}{dp} = \dfrac{pc}{\sqrt{m_0^2c^2+p^2}} = \dfrac{pc^2}{c\sqrt{m_0^2c^2+p^2}} = \dfrac{pc^2}{E}$. Using $E = mc^2$ and $\vec{p} = m\vec{u}$ we arrive at

→ $\boxed{\dfrac{dE}{dp} = u}$

Relativistic dynamics of a single particle → Acceleration of a particle under the influence of a force:-

Conceptually & Classically, force $\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{u}) = m\frac{d\vec{u}}{dt} + \vec{u}\frac{dm}{dt}$.

From $E = mc^2$, $\frac{dE}{dt} = c^2\frac{dm}{dt}$ or $\frac{dm}{dt} = \frac{1}{c^2}\frac{dE}{dt} = \frac{1}{c^2}\frac{d}{dt}(K + m_0c^2) = \frac{1}{c^2}\frac{dK}{dt}$.

As ~~the~~ kinetic energy, $K = \vec{F}\cdot d\vec{l}$. (from Newtonian Mechanics).

$\frac{dK}{dt} = \frac{d}{dt}(\vec{F}\cdot d\vec{l}) = \vec{F}\cdot\frac{d\vec{l}}{dt} = \vec{F}\cdot\vec{u}$ $\therefore$ $\boxed{\frac{dm}{dt} = \frac{1}{c^2}\vec{F}\cdot\vec{u}}$

Next, $\vec{F} = m\frac{d\vec{u}}{dt} + \vec{u}\frac{dm}{dt} = m\frac{d\vec{u}}{dt} + \vec{u}\frac{1}{c^2}\vec{F}\cdot\vec{u} = m\frac{d\vec{u}}{dt} + \frac{\vec{u}(\vec{F}\cdot\vec{u})}{c^2}$

we know that $\frac{d\vec{u}}{dt} = \vec{a}$ $\therefore$ $\vec{F} = m\vec{a} + \frac{\vec{u}(\vec{F}\cdot\vec{u})}{c^2}$ or $\boxed{\vec{a} = \frac{\vec{F}}{m} - \frac{\vec{u}(\vec{F}\cdot\vec{u})}{mc^2}}$

Here, $\vec{F}$ → Newtonian Force &

$\frac{\vec{u}(\vec{F}\cdot\vec{u})}{c^2}$ → Force in Relativity. This means $\vec{a}$ is in the direction of $\vec{u}$ when Force in Relativity i.e $\vec{u} \to \vec{c}$.

P124 last line

Case I:- Force, $\vec{F}$ $\|^{el}$ to velocity, $\vec{v}$. This means $\vec{a}$ is parallel to both $\vec{u}$ & $\vec{F}$.
Particle movement → Straight line. E.g.:- Movement of a charged particle starting from rest in a uniform electric field.
∴ Special case when $\vec{a} \|^{el} \vec{u} \| \vec{F}$ i.e $\vec{a}$ parallel to both $\vec{u}$ & $\vec{F}$.

$\vec{F} = m\left(\frac{du}{dt}\right) + u\left(\frac{dm}{dt}\right)$, using $m = m_0/\sqrt{1-u^2/c^2}$ it can be shown that

$\boxed{\vec{F} = m_0 a/(1-u^2/c^2)^{3/2}}$ → Here, $\vec{F}$ and $\vec{a}$ are parallel to the particle velocity $\vec{u}$.
∴ It can be shown as $\vec{F} = m_0 a/(1-u^2/c^2)^{3/2}$.

Case II ÷ Force, $\vec{F}$ $\|^{el}$ to $\vec{a}$ but $\vec{F} \perp^{r} \vec{u}$ i.e $\vec{F}\cdot\vec{u} = 0$.
using $\vec{a} = \frac{\vec{F}}{m} - \frac{\vec{u}}{mc^2}(\vec{F}\cdot\vec{u})$; Example: $\vec{F} = q\vec{E} + q(\vec{u}\times\vec{B})$, here if $\vec{u} \|^{l} \vec{B}$ then we will only have $\vec{F} = q\vec{E}$. In this case $\vec{F}$ and $\vec{a}$ are perpendicular to the particle velocity $\vec{u}$ i.e $\vec{F}_\perp = \frac{m_0}{\sqrt{1-u^2/c^2}}\cdot a_\perp$ and we have "transverse mass" as $\frac{m_0}{\sqrt{1-u^2/c^2}}$

In Case I ÷ The quantity $\frac{m_0}{(1-u^2/c^2)^{3/2}}$ gives rise to "longitudinal mass".

Q: A particle of charge q starts from rest in an uniform electric field $\vec{E}$. It is made to fall through an electrostatic potential difference of V_0 volts. What is the acquired K.E. by the charge particle?

→ W.d. on the charge q by $\vec{E}$ undergoing a displacement $d\vec{l}$ is $dW = \vec{F}\cdot d\vec{l} = q\vec{E}\cdot d\vec{l}$

Let the uniform field be in the x-direction; $\vec{E}\cdot d\vec{l} = E_x \cdot dx$. and $W = \int q\, E_x \cdot dx$.

Next, $E_x = -(dV/dx)$, where V is the electrostatic potential; such that

$W = -\int q\cdot\frac{dV}{dx}\cdot dx = -q\int dV = -q(V_f - V_i) = q(V_i - V_f) = qV_0$. , where V_0 is the difference between the initial potential V_i and the final potential V_f.

Next, the kinetic energy acquired by the charge is equal to the work done on it by the field,

i.e $K = W = qV_0$.

Note: It is assumed that the charge q of the particle is constant & not dependent on the particle's motion or velocity.

Prb: Let an electron be made to pass through a potential difference of $V_0 = V_i - V_f = -10^4$ volt, i.e a negative charge accelerating in a direction opposite to $\vec{E}$. So, the acquired kinetic energy is $K = qV_0 = eV_0 = (-1.602\times10^{-19})(-10^4)$ joules $= 1.602\times10^{-15}$ joules.

using, $K = mc^2 - m_0c^2$ or $\frac{K}{c^2} = (m - m_0)$ or $(1.602\times10^{-15}\text{ joules}/9\times10^{16}\,m^2/s^2) = m - m_0$

$= 1.78\times10^{-32}$ kg. Consider $m_0 = 9.109\times10^{-31}$ kg, the mass of the moving e^- be $m = (9.109 + 0.178)\times10^{-31}$ kg.

or $m = 9.287\times10^{-31}$ kg.

This gives $m/m_0 = 1.02$ & which means the mass increases due to the motion is about 2 percent of the rest mass. From, $m = m_0/\sqrt{1 - u^2/c^2}$ or $\frac{u^2}{c^2} = [1 - (\frac{m_0}{m})^2] = [1 - (\frac{9.109}{9.287})^2]$

$= 0.038$.

or $u = 0.195c = 5.85\times10^7$ m/s.

This means that the electron acquires a speed of about one-fifth the speed of light; i.e. relativistic prediction.

Q: In a region of uniform magnetic field, a charged particle enters at right angle in the field. This causes the charged particle to move in a circle whose radius is proportional to the particle's momentum. Discuss or show.

→ Charge of particle be q and rest mass, m_0. If the velocity be $\vec{u}$, then the force on the particle is $\vec{F} = q\,\vec{u}\times\vec{B}$ · i.e $\vec{F} \perp^r$ to $\vec{u}$ & $\vec{B}$ (magnetic field).

From, $\vec{a} = \frac{\vec{F}}{m} - \frac{\vec{u}(\vec{F}\cdot\vec{u})}{mc^2}$ → $\vec{a} = \frac{\vec{F}}{m} = \frac{q}{m}\vec{u}\times\vec{B}$ → $\vec{a}$ is in the same direction of $\vec{F}$, but $\perp \vec{u}$.

$\vec{u}$ being constant → Charge particle moves in a circular path of radius (r). → This gives rise to centripetal acceleration $\frac{u^2}{r}$. Next, from above; $a_{centripetal} = a$ from $\frac{q}{m}uB$. i.e

$\frac{q}{m}uB = \frac{u^2}{r}$ or $r = \frac{mu}{qB} = \frac{p}{qB}$. i.e $r \propto p\,(= mu)$.

Prb:- If path of a 10 MeV electron moves at right angle to an uniform magnetic field of strength 2.0 Wb/m², then what is $r_{classical}$ and $r_{relativistic}$?

→ From $r = m_0u/qB$ the classical relⁿ bet^s K & p is $p = \sqrt{2m_0K} = \sqrt{2\times9.1\times10^{-31}\text{ kg}\times10\text{ MeV}\times1.6\times10^{-13}\frac{\text{joule}}{\text{MeV}}}$

$= 17\times10^{-22}$ kg·m/s.

∴ $r = m_0u/qB = p/qB = 17\times10^{-22}/1.6\times10^{-19}\times2 = 0.53$ cm.

Relativistically, $r = mu/qB \Rightarrow p = \frac{1}{c}\sqrt{(K + m_0c^2)^2 - (m_0c^2)^2}$. For rest energy of an e^-, $m_0c^2 = 0.51$ MeV.

∴ $p = \frac{1}{3\times10^8}\sqrt{(10 + 0.51)^2 - (0.51)^2}\;\frac{\text{MeV}\cdot\text{sec}}{m}\,(1.6\times10^{-13}\text{ joule/MeV}) = 5.6\times10^{-21}$ kg·m/s.

$= 5.61\times10^{-21}$ kg·m·s⁻¹.

$\therefore r = \frac{mu}{qB} = \frac{p}{qB} = \frac{5.6\times10^{-21}}{1.6\times10^{-19}\times2.0}\,m = 1.8\,cm.$

Source of e^-s → from the β-decay of radioactive particles.

✓ Bucherer [A.H. Bucherer, Ann. Physik, 28 (1909) p513]. -[1]

✓ W. Bertozzi ["Speed and K.E. of Relativistic Electrons", Am. J. Phys 32 (1964) p551]. -[2]

from Bucherer Results:-

x axis → u/c (Measured)	e/m ($= u/rB$) in coul./kg. (Measured)	e/m_0 ($= \frac{e}{m\sqrt{1-u^2/c^2}}$) in coul/kg. (Computed).
0.3173	1.661×10^{11}	1.752×10^{11}
0.3787	1.630×10^{11}	1.761×10^{11}
0.4281	1.590×10^{11}	1.760×10^{11}
0.5154	1.511×10^{11}	1.763×10^{11}
0.6870 ↓ 0.82	1.283×10^{11}	1.767×10^{11}

The above results are consistent with the relativistic relation $r = \frac{m_0 u}{qB\sqrt{1-u^2/c^2}}$

✓ from the above eqn., $r = \frac{mu}{qB}$; if $q = e$ then $r = \frac{mu}{eB}$ or $\frac{e}{u} = \frac{m}{rB}$ or $\boxed{\frac{e}{m} = \frac{u}{rB}}$.

✓ Using from $r_{classical}$ & r_{Rel} (Relativistic) $\therefore \left(\frac{e}{m}\right)_{Cl.} = \frac{u}{r_{class.}B}$ & $\left(\frac{e}{m}\right)_{Rel.} = \frac{u}{r_{Rel.}B}$.

✓ Relativistic relation, $r_{Rel} = \frac{m_0 u}{qB\sqrt{1-u^2/c^2}}$. or $r_{Rel} = \frac{mu}{qB} = \frac{m_0 u}{qB\sqrt{1-u^2/c^2}}$.

✓ Relativistic electrodynamics, charge of a particle is not changed by its motion → Charge invariance in relativity. Experimentally also confirmed that relativity theory confirms directly the constancy of e.

✓ Bertozzi [W. Bertozzi, "Speed and Kinetic Energy of Relativistic Electrons," Am. J. Phys., 32, p 551 (1964)]

→ Electrons are accelerated to high speed in the electric field of a linear accelerator and emerge into a vacuum chamber. Time of flight in passing two targets of known separation of electrons. Voltage of the accelerator → eV = kinetic energy of the emerging electrons, versus the measured speed $u \equiv K$.

→ "Stopping the electrons in a collector" → K of the absorbed electrons is converted into heat energy which raises the temperature of the collector. To determine the energy released per electron by calorimetry. The average ~~K~~ K per electron, before impact, agrees with the K (eV).

→ X axis: $\frac{2K}{m_0}$ ($10^{16}\,m^2/sec^2$); Y axis: u^2 ($10^{16}\,m^2/sec^2$). At low energies, the experimental results

→ Classical prediction, $K = \frac{1}{2}m_0u^2$ (i.e $2K/m_0 = u^2$). At high energies, $\frac{2K}{m_0} > u^2$.

→ Relativistic prediction, $K = m_0c^2\left[\left(1/\sqrt{1-u^2/c^2}\right) - 1\right]$.

X axis: 0 to 175 ($10^{16}\,m^2/sec^2$). & Y axis: 0 to 12 ($10^{16}\,m^2/sec^2$) → c^2

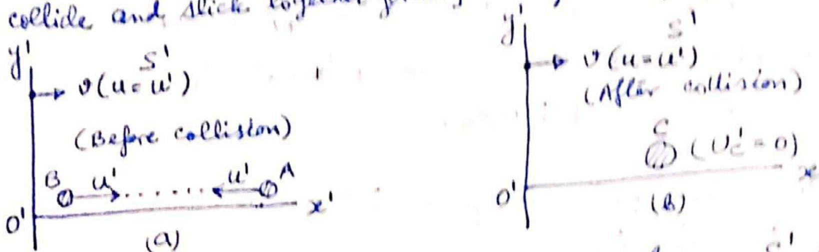
✓ $K = m_0c^2\left(\frac{1}{\sqrt{1-u^2/c^2}} - 1\right)$.

The Equivalence of Mass & Energy:-

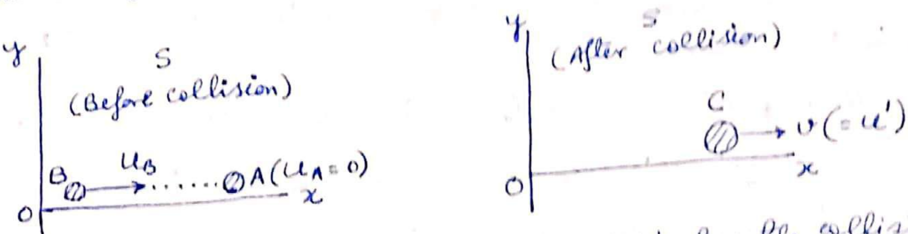
(1) Elastic collision → A collision in which the kinetic energy of the bodies remain constant.

(2) Inelastic collision →

Two identical bodies of rest mass m_o, each with kinetic energy K → seen by observer S', which collide and stick together forming a single body of rest mass M_o.



1) Fig:- A particular inelastic collision as viewed by observer S', (a) before the collision, and (b) after the collision.



2) Fig :- With r.t. 1) fig → As viewed by observer S, (a) before the collision, and (b) after the collision.

The other reference frame S, moving w.r.t S' with a speed $v (= u')$ to the left along the common x-x' axis, the combined body C having the velocity of magnitude v directed to the right along x. Body A is stationary before collision in this frame and body B having speed u_B. The situation in the S-frame. The velocity u_B in the S-frame can be obtained from the relativic velocity transformation eqn., $u_B = \dfrac{u'+v}{1+u'v/c^2} = \dfrac{u'+u'}{1+u'^2/c^2} = 2u'/(1+u'^2/c^2)$. —(1)

The relativistic mass of B in the S-frame, $m_B = \dfrac{m_o}{\sqrt{1-u_B^2/c^2}} = \dfrac{m_o(1+u'^2/c^2)}{(1-u'^2/c^2)}$. —(2)

→ Eqn (2) can be verified.

In the reference frame S, the combined mass C travels at a speed $v (= u)$ after collision as it was stationary in S'. Applying the conservation of relativistic momentum in the x-direction in this frame (the y-component of momentum is inherently conserved), i.e (before) = (after). or

$$\frac{m_o}{\sqrt{1-u_B^2/c^2}}u_B + 0 = \frac{M_o}{\sqrt{1-v^2/C^2}}v$$

Using, $v = u'$ and u_B, we can arrive at

$$\boxed{M_o = \frac{2m_o}{\sqrt{1-u'^2/c^2}} \quad \therefore \boxed{M_o > 2m_o}}$$

So, $M - 2m_o = 2m_o\left(\dfrac{1}{\sqrt{1-u'^2/c^2}} - 1\right)$

Statement :- The rest mass of the combined body is not same as the sum of the rest masses of the original bodies ($2m_o$).

Before the collision of the 2 bodies, the total kinetic energy of the bodies in S' frame equals, $K_A + K_B = 2K = 2m_0c^2\left(\frac{1}{\sqrt{1-u'^2/c^2}} - 1\right)$.

* After collision of the two bodies the final kinetic energy disappears or becomes zero. The energy gets ~~aside~~ converted to internal energy as heat energy or excitation energy.

* Rest mass is equivalent to energy (rest-mass energy) → Application of the conservation of energy principle → Consequence of Lorentz transformation and the conservation of momentum principle.

* From $K_A + K_B = (M_0 - 2m_0)c^2$; the energy associated with the increase in rest mass after the collision, i.e $\Delta m_0 c^2$, equals the kinetic energy present before the collision.

* So, in an inelastic collision kinetic energy alone is not conserved; but the total energy is conserved.

→ * The conservation of total energy is equivalent to the conservation of relativistic mass. This also means that the invariance of energy implies the invariance of (relativistic) mass.

* Mass and energy are equivalent → Formation of a single invariant i.e mass-energy. ∴ $E = mc^2$ → Mass energy, $m = \frac{E}{c^2}$. This means mass can be represented as electron volt.

E.g:- Rest mass of an electron is 0.51 MeV.

* Photons → Particles of zero rest mass → Effective mass equivalent to their energy.